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# Grain marketing gains in Iowa and the use of price forecasting models - a Bayesian decision approach 

 by Hector Eduardo Gonzalez-MendezA Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

Major: Economics

Approved:
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For the Graduate College

> Iowa State University Ames, Iowa

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The prediction of the future and the making of decisions with imperfect knowledge are two of the more complex problems facing Iowa farm operators. A large number of economic models have been developed in recent years which make predictions of future prices. Also, a relatively large amount of theoretical research has been undertaken to single out the uncertainty elements in price prediction. Empirical research, however, has not kept pace with theory by trying to incorporate these elements of uncertainty into the price prediction. The consequence has been that the price predictions of many theoretical models still remain uninfluenced by the advances made in the theory of decisionmaking under uncertainty. This study is primarily empirical in nature and will be confined to the application of decision making analysis under uncertainty to current price forecasting models.

## I. INTRODUCTION

A. Objectives

Prices observed through time are the result of a complex mixture of changes associated with seasonal, cyclical, trend, and irregular factors. The most common regularity observed in agricultural prices is a seasonal pattern of change. Normally, prices of storable commodities are lowest at harvest time and then rise as the season progresses, reaching a peak prior to the next harvest (40, p. 167).

An Iowa farmer does not need to know much about intertemporal economics to realize that the above statement is true in general terms. During the last 26 years (1951-1976) the price per bushel received by farmers in the state of Iowa for corn increased 18 times from December to April; that is, 72 percent of the times, it increased 92.3 percent of the times from April to May, and only 7.7 percent of the times from September to October. A similar phenomenon is observed in the price for soybeans. The price per bushel of soybeans received by farmers in Iowa increased from October to November 77.0 percent of the times during the same years, also, it increased the same number of times, 77.0 percent, from December to January, while it decreased 88.46 percent of the times from August to September. Thus, a farmer who has been in the business long enough is quite aware of the seasonal price movements. The 26 -year averages of these price movements are shown in Figures 1.1 and 1.2

The problems posed to a farmer by the choice between


Figure 1.1. Average monthly corn prices (1951-1976) received by farmers in Iowa (\$/bushel).
(Source: U.S.D.A. Agricultural prices. 1950-1977 annual sumaries) (42)


Figure 1.2. Average monthly soybean prices (1951-1976) received by farmers in Iowa (\$/bushel). (Source: U.S.D.A. Agricultural prices. 1950-1977 annual summaries) (42)
harvest (h) and post-harvest (ph) disposal of his grain are sufficiently important in themselves to be considered explicitly and in detail. The farmer, as a marketing decision maker, has a number of alternative courses of action. These actions are represented by feasible points in time to sell his grain $\left(h, \mathrm{ph}_{1}, \mathrm{ph}_{2}, \ldots, \mathrm{ph}_{i}, \ldots . \mathrm{ph}_{\mathrm{m}}\right.$; where m is sometime prior to the next harvest). The farmer's willingness to retain ownership of the grain during the marketing season depends on the expectation of making larger profit by delaying the marketing transaction.

Current grain prices may be known every day with little uncertainty, but a price to come in the future may not be known that easily. An expected price for grain twelve months from today is less certain than an expected price only one month ahead. Cash prices (CP) in future months are states of nature that can be predicted only with uncertainty. Continuous price changes and their predictability give rise to interesting alternatives for grain marketing during the postharvest season. An expected future price must be, first of all, reliable to the decision maker, and second, it must surpass the harvest price by a margin big enough to compensate at least the carrying charges and the premium for risk taking. Otherwise, the transfer of inventories over time would not be attractive for an economic unit.

In capitalist countries with no price controls the usual
price pattern for a seasonal crop is for the price to rise through the year as a function of the most competitive cost of storing the commodity, including the opportunity cost of capital; for example, Charles C. Cox (7, p. 1216) expresses that ". . . for storable commodities, expected and current spot prices differ only by the net marginal cost of storage." The Neoclassical economic theory states that in a competitive economy, total revenue persistently above total cost will bring more investment into the industry, driving the profit rate down. Thus, if there is a persistent net profit (economic profit) in the grain storage business, we may expect more grain carryover and more investment in storage facilities, which in turn will drive down the net return of the storage activity to a more competitive level with the rest of the economy. Needless to say, the opposite situation will bring about disincentives to grain carryover causing a consequent disincentive to invest in storage facilities. Eventually, the net revenue will go up. It should be borne in mind, therefore, that, under competitive basis, the aggregate farm economic profit of storage must fluctuate around zero over time.

A decision to store grain will be good (or bad) for a farmer if the price differential between the two extremes of the storage period more than (less than) compensates his
carrying charges and premium for risk taking. Usually, for a farmer the storage period begins at harvest time (h) when the market price is $C P(h)$ and ends sometime prior to the next harvest when the price is $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$, $(\mathrm{i} \leq \mathrm{m})$. The farmer's carrying charges and his premium for risk taking are $(C S \& P R)_{f i}$ when he stores from $h$ to $\mathrm{ph}_{i}$. Thus, the net price differential between the harvest and the ith postharvest date is, for the fth farmer,

$$
\begin{equation*}
N P D_{f i}=C P\left(p h_{i}\right)-C P(h)-(C S \& P R)_{f i} i=1, \ldots m \tag{1.1}
\end{equation*}
$$

the subscript $f$ stands for the fth farmer. This net price differential may be decomposed into two fractions; the price differential within the market (market differential) and the price differential from the market (farmer differential), that is,

$$
\begin{equation*}
\mathrm{NPD}_{f i}=P D W M_{i}+\mathrm{PDFM}_{f i} \quad i=1, \ldots \mathrm{~m} \tag{1.2}
\end{equation*}
$$

A price differential within the market, PDWM, is that between the $h$ and $p h$ prices minus the average $C S \& C C^{1}$ prevailing in the agricultural sector of a region (say Iowa);

$$
\begin{equation*}
\operatorname{PDWM}_{i}=C P\left(\mathrm{ph}_{i}\right)-\mathrm{CP}(\mathrm{~h})-\text { Ave. }(\mathrm{CS} \& C C)_{i} \quad i=1, \ldots \mathrm{~m} \tag{1.3}
\end{equation*}
$$

[^0]The $\mathrm{PDWM}_{i}$ 's could all be positive during an entire marketing season which was preceded by a small crop, and all negative if preceded by a large crop (small or large supply compared to the size of the demand). The $P D W M_{i}$ 's would all be zero only if, after a "normal crop" (size), the marketing season demands were correctly anticipated relative to supplies and hence the "correct" quantities were stored between the $\mathrm{ph}_{\mathrm{i}}$ 's and h . In the real world, the occurrence of this last event is highly unlikely.

A price differential from the market, PDFM, is that which arises due to discrepancies between each farmer's intertemporal transfer costs per bushel of grain and those averages for the entire agricultural sector, that is,

$$
\begin{equation*}
P D F M_{f i}=A v e .(C S \& C C)_{i}-(C S \& P R)_{f i} \quad i=1, \ldots m \tag{1.4}
\end{equation*}
$$

If a particular farmer finds himself in a position where either the cost of storage or his opportunity cost for financial resources differ from those prevailing in the entire agricultural sector he may make additional profits or losses. ${ }^{1}$

The farmer's ex-ante choice between harvest and

[^1]post-harvest disposal of his grain depends clearly on the expected net price differentials $\mathrm{NPD}_{\mathrm{fi}}$. The expected $\mathrm{PDFM}_{\mathrm{fi}}$ 's for a coming marketing season can be calculated by each farmer as early as the expected ave. (CS \& CC) ${ }_{i}$ 's are made available to him. It is assumed that he is aware of his (CS \& PR) $f i{ }^{\prime}$.s. The expected $P D W M_{i}$ 's for a season yet to come, however, cannot be estimated that easily by a farmer. They include market elements beyond the control of a farmer; it is here where the market analyst may provide the greatest help to the farming units. The study of expected $P D W M_{i}$ 's is one objective of the present study.

## B. The Price Differentials Within the Market

In most cases individual farmers make their own marketing decisions without the help of a staff of assistants and with the full financial burden of the outcomes falling upon themselves. The development and application of useful marketing decision models could be of extreme importance to farming units in trying to determine the potential gains or losses implied by post-harvest operations. Given that future post-harvest prices are indispensable to estimate the expected $N P D_{f i}$ 's, it is not surprising that farmers are very much interested in the various procedures available to predict prices; to quote Karl A. Fox:

In general, price is the economic variable which farmers are most interested in anticipating. The outlook for various demand and supply factors is of value to farmers mainly, though by no means entirely, as a basis for judging probable trends in prices (12., p. 323).

It is hard to know precisely the degree of sophistication of the methods used by most farming units for predicting future prices, yet there is evidence that they do engage in an analysis of this nature before making their marketing decision (10).

Marketing decision models which correspond to the decision environment and reflect roughly the procedures which farmers can and do use have been developed lately (9). Economists have devoted an enormous amount of time to research attempting to identify empirical regularities in price behavior. Mathematical techniques based on sound theories describe the seasonal components of economic time series and predict to the best of their understanding the price behavior for a future marketing season. Expected $P D_{i}{ }_{i}$ 's can be determined from each mathematical model simply by substituting the model's forecasted prices $\operatorname{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)$ for cash prices $C P\left(\mathrm{ph}_{i}\right)$ in Equation 1.3, that is,

$$
\begin{align*}
\text { Expected }_{P D W M_{i}}= & F P\left(p h_{i}\right)-C P(h)-\text { Ave. }(C S \& C C)_{i} \\
& i=1, \ldots m \tag{1.5}
\end{align*}
$$

If there were a mathematical model capable of reproducing
the entire real world, a farmer would only need to know the model's forecasted prices for a coming marketing season, calculate price differentials within and from the market, and choose for selling his grain the $\mathrm{ph}_{\mathrm{i}}$ (could be a month) where the net differential $N P D_{f i}$ is the largest. Unfortunately: a variety of factors limit the accuracy of postharvest predicted prices. Reliable estimates of the future can be made only insofar as seasonal patterns persist in a uniform manner. External factors suich as government intervention, severe droughts, foreign purchases, etc., create irregular price movements which are unpredictable. Uncertainty elements in nature always influence the outcomes predicted by men. ". . . price expectations reflect information that is neither complete nor perfectly accurate . . ." (7, p. 1218).

Current quantitative methods to predict future prices are many and the fundamental differences among them rely on their initial assumptions. It is not surprising then that the appeal of each to the farming sector cannot be comprehensive. However, there is a degree of correspondence between the initial assumptions of some of these forecasting methods and some of the value judgments which farmers do make in order to assess future prices (15, p. 35). A mathematical model may capture those basic elements that a
farmer year after year takes into consideration, but it is unlikely that it will capture all the peculiarities foreseen (each year) by the farmer as the marketing season begins. Quantitative models designed by economists should be able to introduce in some way these particular seasonal elements as seen by the farm operators.

On the other hand, the power of any method to predict future prices can only be measured by its past performance. A comparison of forecasts and actual outcomes gives us a measure of the accuracy which has been obtained in the past.

To the farmer who acts, or considers acting, on the basis of outlook information, the historical record also offers the most credible evidence as to the level of accuracy which may be expected in the future (12, p. 323).

Attempts should be made to affect the price prediction of a specific forecasting model by its observed effectiveness in the past; in other words, a price forecast should reflect its conditional probability.

The Bayesian Probability Analysis seems to provide the necessary means to determine the expected value of a price forecast after the effectiveness of the model from which it was generated is taken into account, and after the farmer's particular opinions are considered.

## C. The Bayesian Approach

It is, particularly, since Stigler's (38) work on "The Economics of Information" that economists became worried about how individuals should and do behave when imperfectly informed of the consequences of their actions. The introduction of Bayesian Statistics into the economic analysis provided economists with new and powerful tools to cope with the uncertainty elements found in nature. A. E. Baquet, A. N. Halter, and Frank S. Conklin (1) estimated the value of frost forecasting to orchard operators in the context of Bayesian decision making. under uncertainty. Joseph E. Williams and George $W$. Ladd made an application of the Net Energy System and Bayesian Decision Theory to determination of cattle rations and rates of gain (50). The present price analysis is designed to extend existing Bayesian analyses into the grain marketing decision models.

It is clear that a future price is an event whose predictability is neither perfectly possible nor perfectly impossible. Seasonal, cyclical, and trend components allow researchers to define the price pattern, or in other words, allow them to define the feasible range of the outcome. Irregular and external components generate the randomness of the event within the feasible range. Using Bayesian terminology the feasible range can be divided into mutually
exclusive portions called "States of Nature"; $\theta_{j} \varepsilon \theta, j=$ 1,2,...n ( $\theta$ is the feasible range). The marketing season was divided before into $\mathrm{ph}_{\mathrm{i}}$ 's fractions of time (also mutually exclusive; for example, months). The farmer's decision to sell within each time fraction is called "A Decision Maker Action" or simply "Action"; $a_{i} \varepsilon M$, $i=$ $1,2, \ldots, \mathrm{~m}$ ( $M$ is the marketing season). For each actionstate pair an outcome must be named. The outcomes (prices in our case) of all possible pairs provide a set called "Payoff Matrix" of mutually exclusive and collectively exhaustive possibilities of each state of nature and each course of action.

A farmer who observes a forecasted future price $F P\left(p h_{i}\right)$ and expresses his belief that it is either too high or too low does, in fact, assign probabilities to the prediction of the event. These probabilities reflect prior information foreseen by the farmer (perhaps a new government policy) which he thinks is important even though the forecasting model did not take it into account. The farmer's considerations expressed as probabilities in regard to the price forecast FP ( $\mathrm{ph}_{\mathrm{i}}$ ) are introduced in the Bayesian Model as the "prior probability density function."

An econometric forecasting model may be applied repeatedly to past years in order to generate a sample forecast information, that is, the model's predictions of events
which have already occurred. A comparison between cash prices $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$ 's and their predicted values $\mathrm{FP}\left(\mathrm{ph}_{i}\right)$ 's provides a basis for assessing the reliability of the model. As past cash prices occurred inside the feasible range $\theta$ (states of nature domain), any acceptable forecasting model must have made, necessarily its predictions inside the same range or domain. The marginal frequency distribution of price predictions and cash prices (FP \& CP) over the different states of nature are introduced in the Bayesian Model as the "conditional probability density function." Learning then takes the form of changing conditional distributions for the outcomes in future periods on the basis of past outcomes.

The Bayes' Theorem combines both the prior and the conditional probability functions in order to calculate the "Posterior probability density function." The posterior probability density function can be employed to make probability statements about the occurrence of various states of nature, thus, the model's predictions can be improved by applying the posterior probability function to the Payoff Matrix. The Bayesian predictions emanate from previously developed quantitative forecasting models, but in addition, they take into account the elements of the sample forecast information and the farmer's initial information. The importance of the Bayesian approach to this study can hardly
be better expressed than in the words of Richard M. Cyert
and Morris H. DeGroot:

> . . . the term uncertainty takes on a completely different meaning when one moves from classical statistics to a Bayesian approach. Uncertainty has been generally used to describe a situation in which the outcome of the decision maker's action is not precisely predictable because of the existence either of parameters with unknown values or of random error terms. The traditional approach for analyzing uncertainty has been to assume a probability distribution for the error term but not for the unknown values of the parameters (prices). . . . To the Bayesian, all uncertainty can be represented by probability distributions . . ." (8, p. 524).
> We reckon that it is possible to tabulate at the beginning of each marketing season the Bayesian-forecasted prices of a variety of well accepted price forecasting models, taking into account more than one prior probability density function for each model. This possibility gives to the analysis its practical usefulness, since farmers can be in the position to choose, on one hand, the forecasting model (among several) which better resembles the way they make (subjectively or objectively) future price predictions. On the other hand, they can choose the prior probability density function which better reflects their own attitudes towards uncertain elements (elements not captured by the basic assumptions of the model under consideration). With these elements at hand, the model to be developed in the following chapters can provide the farmer with tables of expected

PDWM $_{i}$ 's for the marketing season. The expected $\mathrm{PDWM}_{i}$ 's are the expected gains (or losses) within the market for carrying grain inventories to different points in time. The cost of storage $\mathrm{CS}_{\mathrm{i}}$, the opportunity cost of financial resources $C C_{i}$, and the market prices for grain $C P(h)$ utilized to compute the expected $P D W M_{i}$ 's can be made available to the farmers in order to allow them to calculate their $\mathrm{PDFM}_{f i}$ 's. The ultimate purpose of this thesis will be successfully fulfilled if farmers feel that systematic tabulation of this work does help them in making their marketing decisions.

## II. BASICS ON BAYESIAN DECISION THEORY

The real problems of decision-making under uncertainty are very complicated, so that analysts must reformulate them to something manageable, without losing anything essential to those problems they originally set out to solve. It is the purpose of the next chapter to formulate the grain marketing problems so that they can be solved with the Bayesian Decision Theory. In order to do it intelligently, we must display some knowledge of what Bayesian Decision Theory is, and what it can do; but it is not necessary to have all the details on hand. In this chapter we shall concentrate on the very basic elements of Bayesian Decision Theory and seek to avoid unnecessary detail.

The distinctive features of Bayesian statistics are:
(a) the personalistic interpretation of probability, (it is legitimate to quantify "feelings" about uncertainty in terms of subjectively assessed numerical probabilities) and (b) feasibility of incorporating into the analysis the experiences obtained from sample information. This sample information is expressed in terms of relative frequencies which are the empirical counterparts of theoretical probabilities. Assessments are made of probabilities of events that determine the gains or losses of alternative actions open to a decision maker. The events are assumed to
be limited in number, so are the alternative actions. For each possible action, expected gain or loss, that is a weighted mean of the possible outcomes, can be computed. The weights are nothing but the probabilities of events mentioned above. The action chosen is that one whose expected gain or loss is the largest or the smallest respectively. It can be observed that the principle of decision in Bayesian Decision Theory is the "maximization of expected payoff."

## A. Components of the Bayesian Decision-Making Problem

Strictly speaking the components of the general Bayesian Decision Model are many; however, we limit them to those which are the most relevant in the economic analysis. Based on Paul E. Green and Donald S. Tull (14, p. 39), these components are: (1) the decision maker and his objective; (2) the environment or context of the problem; (3) altérnative courses of action; (4) a set of consequences which relate to courses of action and the occurrence of events not under the control of the decision maker; and (5) a state of doubt as to which course of action is "best." We examine briefly each one of these components. ${ }^{1}$

[^2]1. The decision maker and his objectives

The decision maker may not always be represented by a single individual; however, for simplicity we will assume so hereafter. (In Chapter 3 we will always refer to him as the farmer). He makes rational decisions under uncertainty. By "rational" is meant choice behavior consistent with the assumptions underlying the model--nothing more (R. D. Luce and H. Raiffa, 24). The assumed objectives underlying the model of this dissertation are pecuniaries (e.g., maximize profits or minimize losses).

## 2. The environment of the problem

Problems obviously arise in some type of context. The effectiveness of the decision maker's courses of action will be dependent upon the occurrence of events largely outside of his control, events which he cannot forecast with certainty. These possible events are referred to as "States of Nature," $\theta_{j} \varepsilon \theta$ (i.e., a description of a set of mutually exclusive and collectively exhaustive states $\theta_{j}$ of the decision maker's environment $\theta$ ).
3. Alternative courses of action

A course of action is a specification of some behavioral sequence, such as the sale of a farm product (e.g., grain) at harvest time, the sale of it a month later, etc. All
courses of action involve, either implicitly or explicitly the element of time, although, actions $a_{i}$ can only be taken in the present. "A decision to stipulate a program of action becomes a commitment, made in the present to follow some behavioral pattern in the future" (15, p. 41). The time interval of the courses of action is highly important, inasmuch as both the implicit costs and the probabilities of alternative outcomes will typically vary as a function of time. Courses of action can be spelled out in greater or lesser degree, depending upon the problem under consideration. However, in any case, courses of action are limited in number. For example, a farmer's marketing courses of action can be as many as the number of partitions in his marketing season.

## 4. The consequences of alternative courses of action

The decision maker may be certain about the alternative courses of action but he cannot be certain of the consequences of his choice. It was mentioned before that a primary job is to identify the environment of the problem and consequently to list the states of nature. For each actionstate pair a consequence or outcome $P_{i j}$ must be named. The outcomes of all possible pairs provide a set of mutually exclusive and collectively exhaustive possibilities for each state of nature and for each course of action. The
accuracy of the outcomes attached to the conjunction of each course of action with each state of nature depends upon the conceptualization of the problem faced by the decision maker (the aspect of the conceptualization of the problem provides the reference for its solution). In mathematical terms, for each possible combination of $a_{i}$ and $\theta_{j}$, there is one $P_{i j}$.

The decision problem presented up to here can be illustrated by seeking a maximum $\mathrm{P}_{\mathrm{ij}}$ value (outcome) in payoff Table 2.1. Recall that payoffs reflect the expected monetary values of each outcome deemed possible.

Table 2.1. Conditional payoff table


The three components presented in Table 2.1 plus the decision maker's objective function are all the required ingredients for game theoretic models (R. D. Luce and H. Raiffa, 24) but are only part of the required ingredients of a Bayesian decision model. In the latter model it is assumed that the decision maker is not in "complete ignorance" about (i) the probability distribution of the states of nature of an outcome yet to come, (ii) the long-run frequencies with which these states of nature have occurred in sample information (e.g., time series).

## 5. State of doubt

"To solve a problem is to select some best course of action for attaining the decision maker's objectives." Concerning this component Paul E. Green and Donald S. Tull express that "a state of doubt about which course of action is best can arise under four classes of conditions: 1) certainty with respect to each course of action leading to a specific outcome; 2) risk with respect to each action leading to a set of possible outcomes, each outcome occurring with a known probability; 3) uncertainty with respect to outcomes, given a particular course of action; and 4) partial ignorance with respect to outcomes, given a particular course of action" (14, pp, 44-45). To serve the purpose of this study we concentrate on the last one, here we introduce the
two probability elements of the Bayesian theory which prevent the decision maker from going completely blind into the decision making process.
a. The prior probability distribution "The decision maker is called upon to use his prior judgment (based on his experience and other research) to attribute probabilities to the range of alternatives occurring, given each possible course of action" (G. Wills, 51, p. 180). The prior probability reflects all relevant information concerning various states of the world $\theta_{j}$ before collecting and incorporating sample information into the decision. It is in the prior that we quantify "feelings" about uncertainty in terms of subjectively assessed numerical probabilities. Thus, $P\left(\theta_{j}\right)$ is the prior probability attached by the decision maker to the occurrence of state of nature $\theta_{j}$. One prior probability is assigned to each state of nature $\theta_{j}$ by the decision maker.
b. The conditional probability distribution It is assumed that the decision maker has access to sample information (e.g., historical records). In a way, the decision maker can know which states of nature have occurred under similar decision situations. Also, he can know what states of nature were predicted under those situations. A comparison
of predictions and actual outcomes gives him a measure of the accuracy which has been observed in the sample information. The decision maker is confronted thus with conditional probabilities, that is, with the relative frequency of one forecast $Z_{k}$ given the occurrence of one state of nature $\theta_{j}$. $P\left(Z_{k} / \theta_{j}\right)$ is the conditional probability of observing the $k$ th forecast when the true state of nature is $\theta_{j}$. There is always correspondence between sets $Z$ and $\theta$. Every $Z_{k}$ that belongs to $Z$ is basically a forecast of $a \theta_{j}$ that belongs to $\theta$.

The probability analysis is illustrated in Table 2.2. It summarizes the probability data needed (beyond the game theoretic approach) in the Bayesian approach.

The third type of probability used in Bayesian decision models is the "posterior probability." This probability combines all relevant prior information currently available and sample information. Posterior probability $P\left(\theta_{j} / Z_{k}\right)$ is the probability of observing state of nature $\theta_{j}$ conditional on observing prediction (forecast) $z_{k}$.

The posterior probability $P\left(\theta_{j} / Z_{k}\right)$ is obtained by use of Bayes' Theorem. This theorem combines all relevant information contained in the prior probability density function as given by the decision maker and in the conditional
probability density function, as given by the sample forecast

Table 2.2. Conditional and prior probabilities table

information. The posterior probability density function is calculated by Equation 2.1.

$$
\begin{equation*}
P\left(\theta_{j} / Z_{k}\right)=\frac{P\left(\theta_{j}\right) P\left(Z_{k} / \theta_{j}\right)}{P\left(Z_{k}\right)}, j, k=1, \ldots n \tag{2.1}
\end{equation*}
$$

$P\left(z_{k}\right)$ is the marginal probability of observing the $k$ th outcome of forecast set (Z). It is obtained as in Equation 2.2.

$$
\begin{equation*}
P\left(z_{k}\right)=\sum_{j} P\left(\theta_{j}\right) P\left(z_{k} / \theta_{j}\right), \quad j=1, \ldots n \tag{2.2}
\end{equation*}
$$

The only restrictions placed on the prior, conditional, marginal, and posterior probabilities are that they must be nonnegative and sum up to unity. Failure to comply with these restrictions results in inconsistency.

$$
\begin{array}{ll}
\sum_{j}^{P}\left(\theta_{j}\right)=1 \text { and } P\left(\theta_{j}\right) \geq 0 & \forall j=1, \ldots n \\
\sum_{k} P\left(Z_{k} / \theta_{j}\right)=1 \text { and } P\left(Z_{k} / \theta_{j}\right) \geq 0 & \forall k=1, \ldots n \\
\sum_{k}^{P}\left(Z_{k}\right)=1 \text { and } P\left(Z_{k}\right) \geq 0 & \forall k=1, \ldots n \\
\sum_{j}^{P}\left(\theta_{j} / Z_{k}\right)=1 \text { and } P\left(\theta_{j} / Z_{k}\right) \geq 0 & \forall j=1, \ldots n \tag{2.3d}
\end{array}
$$

## B. Calculating Expected Payoffs under

 the Posterior ProbabilitiesThe final task is to evaluate the expected return of actions. The purpose of this is to norm the criteria of action. The expected return of act $a_{i}$ given a posterior probability density function and subject to observing a decision maker's prediction (forecast) $\mathbf{z}_{k}$ is computed as shown in Equation 2.4 (J. E. Williams, 49, p. 1l).

$$
\begin{equation*}
E P\left(a_{i}\right)_{k}=\sum_{j=1}^{n} P_{i j} \cdot P\left(\theta_{j} / Z_{k}\right) \quad k=1, \ldots n \tag{2.4}
\end{equation*}
$$

The Bayesian strategy if the observed forecasted outcome turns out to be $z_{k}$ is $\left(a_{i}\right)_{k}$ where:

$$
\begin{equation*}
\operatorname{EP}\left(a_{i}^{*}\right)_{k}=\max _{i} \operatorname{EP}\left(a_{i}\right)_{k} k=1, \ldots n \tag{2.5}
\end{equation*}
$$

It must be observed that the value of 2.5 can be computed before the outcome forecast is known. After the forecast observed the selection of the action is possible.

Schematically the process of revising decisions based on prior probabilities and given sample data denoted by conditional probabilities can be represented in a flow diagram. The flow diagram in Figure 2.1 also combines the Bayesian probabilities with the monetary payoffs.


Figure 2.1. Flow diagram of the model used in the Bayesian Decision Theory

## III. THE PRICE BAYESIAN-FORECASTING MODEL

The Iowa State Department of Agriculture, Division of Agricultural Statistics (17, p. 2) defines a farm as "all the land farmed or operated for agricultural purposes by one individual with substantially the same machinery and livestock and with or without the assistance of family and hired labor." The Statistical Reporting Service of the IDA states that the average Iowa farm had about 250 acres in 1976 (17, p. 7). The USDA, ERS-Farm Income Situation Bulletin (44, p. 14) reports that realized annual net income per farm in Iowa has averaged about fifteen thousand dollars during the last five years (1972-1976). A farm with the above characteristics is likely to face liquidity needs for family consumption and growing season expenses which tend to constrain greatly its marketing horizon. Most farmers in Iowa usually do not contemplate marketing alternatives of their products in the long-run (more than one year). Yet, there is a farmer awareness that price changes of farm products give rise to interesting alternatives for grain marketing during the postharvest season. Thus, within some reasonable "short-run," the farmer's willingness to retain ownership of his grain depends on the expectations to make larger profits by delaying the marketing transaction. Profits made purely due to farming activities (harvest time) are not analyzed here; we
are exclusively concerned with "marketing profits" (or losses), that is, those which arise from "marketing actions" after harvest.
A. The Farmer's Perspective

We assume that information such as cash prices, costs of storage (all items), commercial interest rates (on savings and loans), and some general farming background are all available to any marketing decision maker (the farmer). The marketing season (whatever its size) is broken up into discrete intervals $\mathrm{ph}_{\mathrm{i}} \mathrm{i}=1 . . \mathrm{m}$ of equal lengths, ph stands for post-harvest, subscript $i$ stands for the $i t h$ specific marketing period (e.g., quarter, month, or day).

The crop is assumed to be available for marketing immediately after harvest. Producers then have the option of selling any time within the marketing season. Higher prices are required later in the season to induce grain holders to carry it over time. The rise in cash prices must be sufficient to cover the costs of storage and some rate for risk taking. If cash prices are expected to rise by more than carrying expenses, incentives to sell later in the season will increase. If cash prices are expected to rise by less than carrying expenses, farmers' incentives to delay the marketing transaction will decrease. There is a price pattern
which will leave producers indifferent about harvest or postharvest disposal of their grain. This pattern represents the set of prices which exactly compensate storage costs and risk-taking charges. These particular prices are called break-even prices and their computation is as in Equation 3.1.

$$
\begin{equation*}
\operatorname{bep}_{i}=C P(h)(1+r)^{i}+S C_{i} \quad i=1, \ldots m \tag{3.1}
\end{equation*}
$$

Where
bep $_{i}$ - Break-even price for the ith post-harvest period.
CP (h) - Harvest cash price
$r \quad-$ Rate of interest per ph period
$S C_{i}$ - Costs of storage from $h$ to $\mathrm{ph}_{\mathrm{i}}$
Interest rate is introduced in Equation 3.1 under the assumption that producers have an opportunity cost for the money value of the crop.

## B. The Market Perspective

In a market-oriented economy the usual price pattern for a seasonal crop is for the price to rise through the year as a function of the marginal cost of storing the commodity for a competitive industry. Under competitive market conditions the discrepancy between two spot prices of a seasonally-produced commodity represents the marginal revenue of storing it. Economic theory states that the market is
then in equilibrium when the marginal cost of carrying inventories is equal to its marginal revenue ( $16, \mathrm{p} .73$ ).

If future demands for grain were always correctly anticipated relative to supplies and hence the "correct" quantity were stored, the cash prices $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$ would be basically equal to the break-even prices bep ${ }_{i}$. The costs of storage (including the opportunity cost of capital) would be then the only explanatory element for the seasonal price variation. However, future demands are not ever correctly anticipated relative to supplies. Break-even prices are in general the theoretical expectations of the cash prices. Computed bep's (Equation 3.1) for a particular ph period over a number of years are at best the central tendency values of their $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$ 's counterpart, that is,

$$
\begin{align*}
C P\left(p h_{i}\right)_{t}=f\left[\left(b e p_{i}\right)_{t}, U_{i t}\right] \quad & i=\text { post-harvest period }  \tag{3.2}\\
t & =\text { years }
\end{align*}
$$

We think of $U_{i t}$ as a variable with a known probability distribution (over a sample of years) with mean at $\bar{U}_{i t}=$ $\sum_{t=1}^{T} U_{i t} / T$ and having a finite variance $\sigma u_{i}^{2}$. Our approach is largely based on the rational-expectations hypothesis. The concept of rational-expectations was introduced by Muth (28) in 1961. Muth's hypothesis was that mean expectations of firms (farmers in our case) with respect to some phenomenon (say price), was equal to the prediction that would
be made by the relevant economic theory. In this case, the theory in question follows the intertemporal economic approach, whose "best" predictions are the break-even prices. Similar interpretations of the price phenomena have been done thereafter, to quote A. J. Nevins (29, p. 73), "The quantity demanded is assumed to be, in each time period, a random variable with a known probability distribution which depends upon price as a parameter."

The term $U_{i_{t}}$ in 3.2 is referred to as a stochastic disturbance (uncertain) term in most textbooks and so is here. In view of the different nature of the factors involved in the demand and supply forces and their likely independence between marketing seasons an appeal to the central limit theorem would suggest a normal distribution for $\mathrm{U}_{\mathrm{i}_{t}}$, however, at this point this is simply a hypothesis to be proved. It may be that the variable $C P\left(\mathrm{ph}_{\mathrm{i}}\right)_{\mathrm{t}}$ is related to $\left(\operatorname{bep}_{i}\right)_{t}$ in some fashion that when demands are "correctly" anticipated relative to supplies $C P\left(\mathrm{ph}_{\mathrm{i}}\right)_{t}=$ (bep $)_{t}$, yet, there is a basic and unpredictable element of randomness in human responses which can be adequately characterized only by the inclusion of a random variable term $U_{i_{t}}$. This approach to price uncertainty is similar to one followed in inventory models. In studies done by Edwin Mills (26) and Samuel Karlin and Charles Carr (21) the $U$
term is introduced in an additive and in a multiplicative manner, respectively. When uncertainty is introduced in an additive manner, $U$ is a random variable with zero mean. Uncertainty is introduced in a multiplicative manner if $U$ is a random variable with mean that is equal to one.

The additive approach of the stochastic disturbance term $U_{i}$ is the assumption we must follow in order to be consistent with the theory of intertemporal economics (25, pp. 231-296). Thus, the disturbance term has a mean equal to zero for all $\mathrm{ph}_{i}$ and variance increases as $i$ approaches $m$, that is, the variance increases over the marketing season:

$$
\begin{align*}
\mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right)_{t}=\left(\mathrm{bep}_{\mathrm{i}}\right)_{t}+\mathrm{U}_{\mathrm{i}_{t}} \quad & \begin{array}{l}
i
\end{array}=1, \ldots \mathrm{~m}  \tag{3.3}\\
t & =1, \ldots \mathrm{~T}
\end{align*}
$$

Where

$$
U_{i} \sim\left(0, \sigma U_{i}^{2}\right), \sigma U_{i}^{2}>\sigma U_{j}^{2} \text { for } i>j
$$

When the linear hypothesis holds for each $\mathrm{ph}_{\mathrm{i}}$ and the assumptions about disturbance terms holds, too, the price situation over the marketing season can be pictured as in Figure 3.1.

It should be pointed out before leaving this section that the rational-expectations hypothesis is not a necessary condition to the Bayesian approach as such. However, it is essential to the structural form of our model, since the


Figure 3.1. Cash price distributions over a marketing season
bep $_{i}$ analysis does provide the most plausible range of price variations; and consequently, it eliminates from consideration irrelevant alternatives. In conclusion, the rational-expectations hypothesis means fewer states of nature to the Bayesian Decision Model.

## C. The Farmer's Appraisal of the Marketing Problem

One of the main reasons why decision makers do not go blindly into the future is their capacity to predict the behavior of some of the factors involved in the demand and supply forces. The problem of predicting the $C P\left(p h_{i}\right)$ values (prediction of the value itself rather than its expected mean as in the last section) has been the subject of many studies. Expectation models such as the Trend-price Model,
the Moving-Average Model, the Normal-Price Model, etc., have been developed mainly because they appeared to be some of the more logical mechanical models which farmers can use with the knowledge at their command (some of these models will be briefly mentioned later on). Also, economic models which make use of advanced economic analysis have been developed in order to generate more reliable price forecasts, most specialized farm outlook publications base their price predictions on econometric forecasting models of this sort. Especially during the last quarter of the century, the commodity exchanges trading in commodity futures contracts have provided farmers guidance about future price behavior. It is not surprising that many farmers actually use futures markets as reliable sources of price predictions.

It is not simple curiosity that makes farmers interested in future price predictions. On the contrary, accurate price predictions are a basic tool for making marketing profits. According to J. Cantor (5, p. 1), forecasting can be properly defined as "a prediction of a future event with the purpose to converting this estimate into an operating plan." The same.author also believes that "the operating plan that is developed based on the forecast is the most important consideration." On view of this definition, it is apparent that farmers search for price forecasts $F P\left(\mathrm{ph}_{i}\right)$, which predict
future cash prices $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$, in order to specify an operating plan of grain marketing.

Once a farmer has made up his mind about the set of most reliable price forecasts, he has two immediate steps to follow: (1) compute the break-even prices which correspond to the price forecasts, that is, calculate bep ${ }_{i}$ for each $\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)$; (2) determine expected marketing gains (or losses) by subtracting bep from $_{i}$ FP ( $\mathrm{ph}_{\mathrm{i}}$ ) for all $i$ (notice that we calculate here the expected gain within the market - PDWM ${ }_{i}$ since we deal only with average figures). Chances are that a farmer who follows these steps will be more inclined to store his grain if he finds that $F P\left(\mathrm{ph}_{\mathrm{i}}\right)>$ bep $_{i}$ at least for some i's. So, basically the problem reduces to: "Compare bep ${ }_{i}$ with $F P\left(\mathrm{ph}_{\mathrm{i}}\right)$ for all i." The proposition may seem attractive, but there is little to conclude from it. The price forecasts in question are not certain whatsoever. A farmer, as a decision maker, may find it valuable to ask such questions as the following: How were the price forecasts generated?; did they take into account all the relevant variables from the farmer's point of view?; is there any reliable theory behind them?; if so, how has this theory forecasted in the past?; what about other alternative forecasting methods? In other words, we have said nothing in regard to the effectiveness of the price forecasting method in question, nor have we mentioned its capabilities to incorporate
changing conditions into the parameters or into the initial assumptions of its price analysis. Probabilistic information and degrees of dispersion of specific variables are needed. To this we now turn.

## D. Defining the States of Nature. of Cash Prices

It was said before that a future price is an event whose predictability is not perfectly possible; yet seasonal, cyclical and trend components allow us to define the feasible range of the outcome. The measure of central tendency of the feasible range is by hypothesis the set of bep ${ }_{i}$ prices (for the same $\mathrm{ph}_{\mathrm{i}}$ over a sample of years). This hypothesis provides a partial summary of the information contained in seasonal price data. The need for a measure of variation of the feasible range is apparent. We need to make sure that events outside the feasible range of the outcome have either very little or no statistical probability of occurrence; so that, their inclusion in the analysis will be not only cumbersome, but meaningless too.

Based on the central limit theorem and on the rationalexpectations hypothesis, we assumed that $\left[C P\left(\mathrm{ph}_{i}\right)-\mathrm{bep}_{i}\right]_{t}$ has a known probability distribution with mean zero and variance $\sigma U_{i}{ }^{2}$. If the sample mean has approximately a normal distribution due to the randomness of demand and supply forces to
price formation, the normal distribution is a continuous, symmetrical, bell-shaped probability distribution, as assumed by Equation 3.3. If the random variable $C P\left(\mathrm{ph}_{\mathrm{i}}\right)_{t}$ has a normal probability distribution over its values of central tendency, then $C P\left(\mathrm{ph}_{\mathrm{i}}\right)_{t}$ is a normal variable itself, that is, its probability density function would be:

$$
P\left\{C P\left(p h_{i}\right)-\text { bep }_{i}\right\}=\frac{1}{\sqrt{2 \pi} \sigma U_{i}} e^{-\frac{1}{2}\left\{\frac{C P\left(p h_{i}\right)-\text { bep }_{i}}{\sigma U_{i}}\right\}^{2}}
$$

To simplify the analysis at this point, let us assume the $\mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right)_{t}$ distribution is normal about its values of central tendency $\left(\operatorname{bep}_{i}\right)_{t}$. Assume a situation at the beginning of the marketing season, the $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$ price has not yet been observed. The theoretical expected mean (bep ${ }_{i}$ ) of the particular outcome has been estimated and using the variance $\sigma U_{i}{ }^{2}$ the probability distribution of it has been determined. The case is depicted in Figure 3.2 with a normal distribution density function. The distribution is centered at the value bep $_{i}$.

Notice that we have dropped the subscript $t$ from the notation as we refer to one marketing season. The magnitude of the area A in Figure 3.2 gives the probability that the random event called cash price will lie between $C P\left(\mathrm{ph}_{\mathrm{i}}\right) 1$ and $\mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right) 2$; that is, $\mathrm{P}\left[\mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right) 1 \leq \mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right) \leq \mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right) 2\right]=$


Figure 3.2. The cash prices normal distribution for the ith post-harvest period

Area of A. The total area under the curve and above the $\mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right)$ axis is one, as required by Equation 2.3. A partition to the continuous normal distribution can be made in order to define a number of discrete intervals, all equally likely, mutually exclusive, and collectively exhaustive of the probability distribution of the random event. The normal probability density function in Figure 3.3 shows five intervals of this sort.


Figure 3.3. Five states of nature within the feasible range of the random event $C P\left(\mathrm{ph}_{i}\right)$

Assuming that areas $A, B, C, D$, and $E$ are equivalent, the probabilities of the mutually exclusive intervals on the axis are equal. We define these intervals as the states of nature of the future cash price $C P\left(\mathrm{ph}_{i}\right)$, that is, $\theta_{j i}$ is the state of nature $j$ in the ith post-harvest period. States of nature are comprehensive of the entire probability universe; $\theta_{j i} \varepsilon \theta$ ( $\theta$ is the feasible range of the outcome, say defined by a $95 \%$ level of significance). Using the probability tables of the appropriate distribution we define the border values between two states of nature. Table 3.1 shows five states of nature of this sort.

Table 3.1. States of nature of expected cash prices

| States of | States of nature of expected cash price for the |
| :--- | ---: |
| nature $\theta_{j i}$ | ith post-harvest marketing period |

${ }^{\theta}{ }_{1 i} \quad$ bep $_{i}+t_{.215} \sigma U_{i} \leq C P\left(p h_{i}\right) \leq \operatorname{bep}_{i}+t .025^{\sigma U_{i}}$
$\theta_{2 i} \quad$ bep $_{i}+t_{.405} \sigma U_{i} \leq C P\left(p h_{i}\right) \leq$ bep $_{i}+t .215 \sigma U_{i}$
$\theta_{3 i} \quad \operatorname{bep}_{i}-t_{.405} \sigma U_{i} \leq C P\left(p h_{i}\right) \leq \operatorname{bep}_{i}+t .405 \sigma U_{i}$
$\theta_{4 i} \quad \operatorname{bep}_{i}-t_{.215} \sigma U_{i} \leq C P\left(\mathrm{ph}_{i}\right) \leq \operatorname{bep}_{i}-t .405 \sigma U_{i}$
$\theta_{5 i} \quad$ bep $_{i}-t .025 \sigma U_{i} \leq C P\left(p h_{i}\right) \leq \operatorname{bep}_{i}-t .215 \sigma U_{i}$
E. Defining the Price Forecast Intervals

A process which chooses at random a real number between zero and ten is likely to make a "better" prediction of the price per bushel of corn for the next April the first than a similar process which chooses a random number between zero and one thousand. A third process which chooses, still at random, but only inside "some" feasible range of the price in question will most likely make a better prediction than either of the former two. Most forecasting and expectation models are similar to the third process in the sense that they predict only inside some feasible range for the event,
but unlike all processes above, they also try to remove at least some of the randomness of the price-selection process. Common sense tells us that we should be concerned only about those which fall in the latter category.

Current price-expectation models and econometric forecasting models are numerous. Each model may generate a set of price forecasts $\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)$ 's. A break-even price obtained from Equation 3.1 is the result of a mathematical identity which defines the expected mean of the outcome. Price forecasting models are functional forms which make predictions about the outcome itself, independently of its expected mean. In fact, this is what makes a price forecasting model worth while. Going back to our initial notation, we can state that;

$$
\begin{equation*}
F P\left(\mathrm{ph}_{i}\right)_{t}=\left(\operatorname{bep}_{i}\right)_{t}+\varepsilon_{i_{t}} \tag{3.5}
\end{equation*}
$$

The assumptions behind the $\varepsilon_{i_{t}}$ term cannot be the same as those of the $U_{i_{t}}$ term in Section 3.2. The prediction value does not contain by definition the random elements embodied in the actual outcomes. Nevertheless, more accurate price predictions are expected as the $\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)_{t}$ distribution of a forecasting model approximates to the probability distribution of the cash prices $C P\left(\mathrm{ph}_{\mathrm{i}}\right)_{t}$. In other words, the biasness factor $\bar{\varepsilon}_{i}=\sum_{t=1}^{T} \varepsilon_{i_{t}} / T$ must not be significantly different from
zero, and the variance of the $F P\left(\mathrm{ph}_{\mathrm{i}}\right)_{t}$ distribution must approximate to the variance of the distribution of the means. A plausible situation, as seen at the beginning of a marketing season is depicted in Figure 3.4 (we assume normality of the distributions only to simplify the graphical form).


1. $\sigma U_{i}$
2. $\sigma F P\left(p h_{i}\right)$
3. Feasible range of the outcome
4. Feasible range of the forecast

Figure 3.4. Hypothetical illustration of $\mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right)$ and $\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)$ distributions

A feasible range of the forecast wider than the feasible range of the outcome would mean that the price forecasting model can make predictions which have no theoretical support, that is, predictions which are irrelevant alternatives from the probabilistic point of view. The opposite situation would mean, of course, that a number of relevant alternatives are
overlooked systematically by the forecasting model.
The model in this thesis is bounded to elaborate on the alternatives which have a relevant probability. Thus, we restrict the feasible range of the forecast to the dimension of the feasible range of the outcome.

Following the same approach of last section, the probability density function $\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)$ is divided into five discrete intervals which are equally likely, mutually exclusive, and collectively exhaustive of the feasible range of the forecast. The intervals of the forecast resemble the states of nature of the outcome, as shown in Table 3.2.

Before leaving this section, notice that a "perfect" forecasting model means that $\varepsilon_{i_{t}}=U_{i_{t}}$ for all post-harvest periods $i$ and for all marketing seasons $t$; thus, a linear regression model $\mathrm{U}_{\mathrm{i}_{t}}=\alpha+\beta \varepsilon_{i_{t}}$ (holding $i$ constant and letting $t$ to vary) will indicate the degree of effectiveness of a forecasting model. The regression coefficients $\alpha$ and $\beta$ approach zero and one and the coefficient of determination $R^{2}$ approaches one as we move from less to more accurate forecasting models.

Table 3.2. Price forecast intervals of the feasible set
Price forecast Intervals of the feasible range of the forecast intervals $\mathrm{Z}_{\mathrm{ki}}$ for the ith post-harvest period

$$
\begin{aligned}
& Z_{1 i} \quad \text { bep }_{i}+t_{.215} \sigma U_{i} \leq F P\left(p h_{i}\right) \leq \text { bep }_{i}+t .025 \sigma U_{i} \\
& \mathrm{Z}_{2 i} \quad \mathrm{bep}_{i}+t_{.405} \sigma \mathrm{U}_{\mathrm{i}} \leq \mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right) \leq \mathrm{bep}_{\mathrm{i}}+t_{.215} \sigma \mathrm{U}_{\mathrm{i}} \\
& \mathrm{Z}_{3 i} \quad \text { bep }-\mathrm{t}_{.405} \sigma \mathrm{U}_{\mathrm{i}} \leq \mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right) \leq \operatorname{bep}_{\mathrm{i}}+\mathrm{t}_{.405} \sigma \mathrm{U}_{\mathrm{i}} \\
& Z_{4 i} \quad \operatorname{bep}_{i}-t_{.215} \sigma U_{i} \leq F P\left(\mathrm{ph}_{i}\right) \leq \operatorname{bep}_{i}-t_{.405} \sigma U_{i} \\
& Z_{5 i} \quad \operatorname{bep}_{i}-t .025 \sigma U_{i} \leq F P\left(\mathrm{ph}_{i}\right) \leq \operatorname{bep}_{i}-t .215 \sigma U_{i}
\end{aligned}
$$

F. Defining the Farmers Prior Probability Distributions over a Set of Cash Prices (Data and Nondata Priors)

The importance of exercising care and thought in choosing a prior probability density function $P\left(\theta_{j i}\right)\left(\theta_{j i}\right.$ is state of nature $j$ in the ith post-harvest period) to represent prior information should be pointed out. The prior information may arise from introspection, casual observation, or even some "theoretical" considerations of the decision maker. The farmer, as a decision maker, calls upon his prior information (based on his experience and other research) to attribute probabilities to the five states of nature. When a prior
probability density function represents information obtained subjectively or by casual observation, it is known in statistics as a "nondata-prior." It is distinguished from the prior based on information contained in samples of past data. This kind of prior is known as "data-prior" and will be obtained from the relative frequency distribution of past predictions over the states of nature.

In any case, the prior probability density function becomes the probability distribution assigned by a decision maker to the states of nature $\theta_{j i}$. It must be noted that one person's prior can differ from that of another, even if both are confronted with very similar or identical circumstances. For example, prior distribution about the mean bep ${ }_{i}$ can be thought to be normal by one, skewed to the right by another, and skewed to the left by a third, as shown in Figure 3.5.

Decision maker's prior probability distribution represented by Figure 3.5 (a) has a mean $\mu_{\theta i}=$ bep $_{i}$ (e.g., it can be that either forecast information does not have influence on him while theoretical information does, or central values have strong appeal to him). Decision maker's prior probability distribution represented by Figure 3.5(b) has a prior mean $\mu_{\theta i}>$ bep $_{i}$; he may be an optimistic person by nature, he assigns higher probabilities to higher payoff


Figure 3.5. Hypothetical prior probability distribution over the states of nature $\theta_{j i}$
values and lower probabilities to lower payoff values. The prior probabilities represented by Figure 3.5(c) have a prior mean $\mu_{\theta i}$ < bep ${ }_{i}$, that is, decision maker behaves more pessimystically in the process of assigning probabilities to each state of nature. It must be pointed out that the decision maker's distribution may be totally or partially founded on experience, or even lack it. Some priors may imply the use of "some sort of data." Others may not. It is extremely difficult to formulate general precepts regarding the appropriate "prior" used by farmers, since much depends on their degree of objectivity.

Prior probability distributions $P\left(\theta_{j i}\right)$ 's may not be necessarily bell-shaped. Linearly increasing, decreasing, and uniform probability distributions are conceivable kinds, too. Also another kind may be thought up by the farmer as well.
G. Defining the Conditional Probability Distribution of a Price Forecasting Model

The problem of conditional probability is that of finding the probability of one event given the occurrence of another event. In terms of the events defined earlier, we want to find out from sample data the frequency with which price forecasts $F P\left(p h_{i}\right) \varepsilon Z_{k i}\left(Z_{k i}\right.$ is the forecast interval $k$ in the ith post-harvest period) occurred given that
outcomes $C P\left(\mathrm{ph}_{\mathrm{i}}\right) \varepsilon \theta_{j i}$ occurred. In other words, states of nature $\theta_{j i}$ 's have been observed in past years, also a set forecasts $\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)$ 's belonging to $\mathrm{Z}_{\mathrm{ki}}$ 's were made before the outcomes were observed. A comparison of the past forecasts and past outcomes gives us a relative frequency distribution of $Z_{k i}$ 's over $\theta_{j i}$ 's. The conditional probability $P\left(Z_{k} / \theta_{j}\right)_{i}$ is defined consequently as the fraction of times the forecast $Z_{k i}$ occurred in a sizable series of observations $\theta_{j i}$ for any $i$ common to both events. As represented in a Venn diagram, the problem is to find the ratio of the area $\left(z_{k} \theta_{j}\right) \frac{1}{i}^{1}$ to the area of $\left(\theta_{j}\right)_{i}$ (Figure 3.6). Since $\theta_{j i}$ has occurred, it is known that the outcome is a point in the circle $\theta_{j}$. If forecast $Z_{k i}$ was issued, the outcome is one of the points within $\theta_{j}$. which is also within $Z_{k}$.


Figure 3.6. Conditional probability Venn diagram
${ }^{1}$ Subscript $\bar{i}$ means holding $i$ constant.

Accordingly, $P\left(Z_{k} / \theta_{j}\right)$ is a probability set function, defined for subsets of $\theta_{j_{i}}$. It may be called the conditional probability set function, relative to the state of nature $\theta_{j}$ for the $i$ th period of the marketing season and when $z_{k} \varepsilon Z$ (the feasible set of the forecast) and $\theta_{j} \varepsilon \theta$ (the feasible set of the outcome).

As a farmer's information relating to particular future price changes, he revises his estimates of probabilities of the various prices in consideration. The process of revising probabilities associated with expected prices in the face of new information is the essence of learning from experience. The process of revising probabilities representing degrees of belief in expected prices to incorporate new information can be made operational and quantitative by use of the rule of probability theory named Bayes' Theorem as explained in Chapter II. Now, the prior probability $P\left(\hat{o}_{j i}\right)$ is combined with the conditional probability $P\left(Z_{k} / \theta_{j}\right)_{i}$ by means of Bayes' Theorem to yield the posterior probability $P\left(\theta_{j} / Z_{k}\right)_{i} ; \quad i=1, \ldots m$ and $j, k=1, \ldots n$. The posterior probability is seen to depend then on both the prior information used by farmers and the time-series sample information provided by the price forecasting method, one is as crucial as the other. The posterior probability density function is calculated from Bayesian Equation 3.6.

$$
\begin{equation*}
P\left(\theta_{j} / z_{k}\right)_{i}=\frac{P\left(\theta_{j i}\right) P\left(z_{k} / \theta_{j}\right)_{i}}{P\left(z_{k i}\right)} \quad i=1, \ldots m \tag{3.6}
\end{equation*}
$$

$P\left(Z_{k i}\right)$ is the marginal probability density function of observing a forecast within the kth interval of the feasible range 2 for the $i$ th period of the marketing season. It is obtained from Equation 3.7.

$$
\begin{equation*}
P\left(z_{k i}\right)=\sum_{j=1}^{n} P\left(\theta_{j i}\right) P\left(z_{k} / \theta_{j}\right)_{i} \quad i=1, \ldots m \tag{3.7}
\end{equation*}
$$

The posterior probability d.f. $P\left(\theta_{j} / Z_{k}\right)_{i}$ can be employed to make probability statements about the state of nature $\theta_{j i}$; for example, to compute the probability that $\theta_{j i}<C P\left(\mathrm{ph}_{\mathrm{i}}\right)<$ $\theta_{h i}$, where $j \neq h$. If $P\left(z_{k} / \theta_{j}\right)_{i} k=j$ approaches one as the sample information grows (for every i) in past data, the conditional probability will more and more dominate the prior probability in determination of the posterior probability. The latter will become more concentrated about the "true" value of the parameter. Moreover, if two farmers have different nondataprior probability vectors, perhaps because they have different initial information, their posterior probability density functions will become similar as additional common data is combined with their respective priors. If $P\left(Z_{k} / \theta_{j}\right)_{i}$ is near zero for $k=j$ in a sizable sample, it should be concluded that $z$ distribution (for every i) is not a good forecast of $\theta$ distribution (for the same i). The prior dis-
tribution becomes then the most valuable probability information. This may raise interesting questions of performance of two or more forecasting methods when analyzed under the same and under different sample conditions.

## H. The Prediction Posterior Probabịlity Distribution

There are at least two ways to arrange the discrete probability elements contained in the posterior probability function $P\left(\theta_{j} / Z_{k}\right)_{i}$. For each value of $i$, one can define an array of $n \times n$ elements since $j, k=1,2, \ldots$. . Table 3.3 shows the case assuming five states of nature and five forecast intervals. In general, the number of elements in each array depends on the partition of the feasible range of the outcome and the feasible range of the forecast.

Table 3.3. Posterior probability for the ith marketing period (assuming that five states of nature and five forecast intervals have been defined) $P\left(\theta_{j} / Z_{k}\right){ }_{i=\text { const }}$.

| States of nature | Forecast Intervals |  |
| :---: | :---: | :---: |
|  | $\mathrm{Z}_{1 i} \quad \mathrm{Z}_{2 i}$ | $\mathrm{Z}_{3 i} \quad \mathrm{Z}_{4 i} \quad \mathrm{Z}_{5 i}$ |
| ${ }^{\theta} 1 \mathrm{i}$ | $\mathrm{P}\left(\theta_{1} / Z_{1}\right) \overleftarrow{i} \mathrm{P}\left(\theta_{1} / z_{2}\right) \bar{i}$ | $P\left(\theta_{1} / Z_{3}\right)-P\left(\theta_{1} / Z_{4}\right)-\mathrm{P}\left(\theta_{1} / Z_{5}\right) \bar{i}$ |
| ${ }^{(2 i}$ | $\mathrm{P}\left(\theta_{2} / Z_{1}\right) \bar{i}^{P}\left(\theta_{2} / Z_{2}\right) \bar{i}$ | $\mathrm{P}\left(\theta_{2} / Z_{3}\right) \bar{i} \mathrm{P}\left(\theta_{2} / Z_{4}\right) \bar{i} \mathrm{P}\left(\theta_{2} / Z_{5}\right) \bar{i}$ |
| ${ }^{3 i}$ | $P\left(\theta_{3} / z_{1}\right) \bar{i} P\left(\theta_{3} / z_{2}\right) \bar{i}$ |  |
| ${ }^{4} \mathrm{i}$ | $\mathrm{P}\left(\theta_{4} / \mathrm{Z}_{1}\right) \bar{i} \mathrm{P}\left(\theta_{4} / \mathrm{Z}_{2}\right) \bar{i}$ | $\mathrm{P}\left(\theta_{4} / z_{3}\right)$ - $\mathrm{P}\left(\theta_{4} / Z_{4}\right) \bar{i} \mathrm{P}\left(\theta_{4} / Z_{5}\right) \overline{\mathrm{i}}$ |
| ${ }^{\theta} 5$ | $\mathrm{P}\left(\theta_{5} / Z_{1}\right) \bar{i} \mathrm{P}\left(\theta_{5} / Z_{2}\right) \bar{i}$ | $\mathrm{P}\left(\theta_{5} / Z_{3}\right) \overline{\mathrm{i}} \mathrm{P}\left(\theta_{5} / Z_{4}\right) \overline{\mathrm{i}} \mathrm{P}\left(\theta_{5} / Z_{5}\right) \overline{\mathrm{i}}$ |
|  |  |  |

By the same token, holding $k$ constant and letting $i$ to vary, new nxm arrays can be defined. Clearly as $k$ goes from one to $n, n$ arrays of this sort are possible. We call arrays of this sort the "Prediction Posterior Probability Matrices" $\mathbb{P} \mathbb{P} \mathbb{P}_{k}$, they are seen to depend on one single price forecast interval. In other words, $P\left(\theta_{j} / Z_{k=c o n s t .}\right)_{i}$ combines only one price forecast interval $k$ with all the states of nature and all the post-harvest periods. Table 3.4 illustrates one example (when $n=5$ ) of a prediction posterior probability matrix for the $k$ th forecast interval, $\mathbb{P} \mathbb{P}_{\mathbb{P}}$.

Table 3.4. Prediction posterior probability matrix for the kth forecast interval (assuming five states of nature) $P\left(\theta_{j} / Z_{k=\text { const. }}\right)_{i}$

| States of nature | $\overline{\mathrm{ph}_{7}}$ | Post-Harvest Periods <br> ph |
| :---: | :---: | :---: |
|  | $\mathrm{ph}_{1}$ | $\mathrm{ph}_{2}$ |
| $\theta_{1}$ | $\mathrm{P}\left(\theta_{1} / Z_{k}\right)_{1}$ |  |
| ${ }^{2}$ | $\mathrm{P}\left(\theta_{2} / \mathrm{Z}_{\mathrm{k}}\right)_{1}$ | $P\left(\theta_{2} / Z_{k}\right)_{2} \cdot . . . . . . P\left(\theta_{2} / Z_{k}\right)_{\mathrm{m}}$ |
| $\theta_{3}$ | $\mathrm{P}\left(\theta_{3} / z_{\mathrm{k}}\right)_{1}$ | $P\left(\theta_{3} / Z_{k}\right)_{2} \cdot \cdots . . . . P\left(\theta_{3} / Z_{k}\right)_{m}$ |
| ${ }^{*} 4$ | $P\left(\theta_{4} / z_{k}\right)_{1}$ |  |
| $\theta_{5}$ | $P\left(\theta_{5} / Z_{k}\right)_{1}$ | $\mathrm{P}\left(\theta_{5} / \mathrm{Z}_{\mathrm{k}}\right)_{2}$. . . . . . $\mathrm{P}\left(\theta_{5} / \mathrm{Z}_{\mathrm{k}}\right)_{m}$ |

## I. Alternative Courses of Action and the Payoff Matrix

A course of action is a specification of some behavioral sequence. The farmer, as a marketing decision maker, has a number of alternative courses of action. These actions are represented here by the possibilities of selling his grain at harvest or within each of the post-harvest periods. The farmer's decision to sell in the $\mathrm{ph}_{\mathrm{i}}$ period is called the decision maker action $a_{i}$. It is assumed then that the farmer is certain about the $m$ ( $i=1, . . . m$ ) alternative courses of action. The environment of the problem was identified in Section 3.4 and five states of nature were determined per $\mathrm{ph}_{\text {ith }}$ period. For each action-state pair a consequence or outcome must be named. The mid-range values $P_{i j}$ of the intervals depicted in Figure 3.3 define the outcomes of the states of nature for the $\mathrm{ph}_{\mathrm{i}}$ period. The outcomes of all possible combinations (pairs) provide a set of mutually exclusive and collectively exhaustive possibilities for the feasible ranges of the post-harvest periods and for the periods themselves. The set so defined is called payoff matrix. The elements in the payoff matrix are prices per unit of grain associated to states of nature and actions. Table 3.5 depicts the mxn array of the $P_{i j}$ elements of the payoff matrix.

Table 3.5. Payoff matrix per bushel of grain

| Courses of <br> action | States of Nature | $\theta_{1} \quad \theta_{2}$ | $\theta_{5}$ |
| :--- | :--- | :--- | :--- |


| $a_{1}$ | $P_{11}$ | $P_{12}$ | $\cdot$ | $\cdot$ | $\cdot$ | $P_{15}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $a_{2}$ | $P_{21}$ | $P_{22}$ | • | • | • | $P_{25}$ |


mxn

## J. Computing Bayesian Strategies

A Bayesian strategy is the selection of the specific course of action which maximizes the weighted mean-price of the possible outcomes. The weights are taken from a given probability distribution over the state of nature. Two types of Bayesian strategies can be produced in Bayesian decision models. One utilizes exclusively the prior
probability density function $P\left(\theta_{j i}\right)$ (this can either be a "data-prior" or a "nondata prior") and the other utilizes the posterior probability density function $P\left(\theta_{j} / Z_{k}\right)_{i}$. In Bayesian theory the former is referred to as the "NONDATA" approach, and the latter is referred to as the "DATA" approach. ${ }^{1}$

Under the NONDATA approach, the expected price of an act $a_{i}$ is $\operatorname{ep}\left(a_{i}\right)$, this is computed as shown in Equation 3.8.

$$
\begin{equation*}
e p\left(a_{i}\right)=\sum_{j=1}^{n} p_{i j} \cdot P\left(\theta_{j i}\right) \quad i=1, \ldots m \tag{3.8}
\end{equation*}
$$

The NONDATA Bayesian strategy is to select course of action $a_{i}$ * such that the expected gain e.g., expected price minus its break-even price, is maximized.

$$
e g^{*}\left(a_{i}^{*}\right)=\max _{i}\left\{\operatorname{ep}\left(a_{i}\right)-\operatorname{bep}_{i}\right\} \quad i=1, \ldots m
$$

Under the DATA approach, the expected price of an act $a_{i}$ given a posterior probability d.f. and a price forecast $\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)$ which belongs to a $\mathrm{z}_{\mathrm{k}}$ interval is computed as shown in Equation 3.10.
${ }^{1}$ Capital letters are used exclusively for the "NONDATA" and "DATA" Bayesian approaches to avoid confusion with the "nondata" and "data" priors.

$$
e p\left(a_{i} / Z_{k}\right)=\sum_{j=1}^{n} P_{i j} \cdot P\left(\theta_{j} / Z_{k}\right)_{i} \quad \begin{align*}
& i=1, \ldots m  \tag{3.10}\\
& k=1, \ldots n
\end{align*}
$$

The DATA Bayesian strategy is to select course of action $a_{i}^{* *} / Z_{k}$ * once the relevant price forecasts $F P *\left(p h_{i}\right) \varepsilon Z_{k}$ * for the $\mathrm{ph}_{\mathrm{i}}$ periods are known. In our case, there are five $Z_{k}$ 's per $p h_{i}$ but only one $Z_{k *}$ per $p h_{i}$, one $a_{i}^{*} / Z_{k}$ * per $Z_{k} *$ ' and one $a_{i}^{* *} / z_{k} *$ per marketing season. The course of action selected must maximize expected gain eg( $a_{i}^{* *} / Z_{k^{*}}$ ) : expected price conditional to observed $\mathrm{Z}_{\mathrm{k}}$ * forecast minus break-even price as illustrated in Equation 3.11.

$$
\begin{equation*}
\operatorname{eg}\left(a_{i}^{*} * / Z_{k *}\right)=\max _{i}\left\{e p\left(a_{i}^{*} / z_{k^{*}}\right)-\operatorname{bep}_{i}\right\} i=1, \ldots m \tag{3.11}
\end{equation*}
$$

where $k *$ 's are the intervals of the relevant set of forecasts FP* $\left(\mathrm{ph}_{\mathrm{i}}\right)$. Notice also that the values of 3.10 can be computed before the set of forecasts $F P *\left(p h_{i}\right) \varepsilon Z_{k}$ * is known. The NONDATA approach with nondata priors is based on information entirely independent of the forecasts, it is based on probabilities attributed by the farmer to the states of nature in each particular post-harvest period. The NONDATA approach with data priors is based on forecast information but with no reference to its incidence on actual outcomes. The DATA approach with data priors is tautologic by definition. The DATA approach with nondata priors is based on the farmer's "feelings" and on the past performance
of a price forecasting model. The number of alternative solutions from "data" and "nondata" priors and "DATA" and "NONDATA" approaches increases as we wish to consider; on one hand several forecasting models, and on the other, alternative nondata priors. These aspects of the decision problem create a wide spectrum of possibilities which demand (for the sake of clarity) a more systematic approach. 1. The NONDATA approach regardless of the nature of the prior The computational procedure to obtain a NONDATA Bayesian strategy requires a set of prior distributions (data or nondata) and the payoff matrix. The nature of decision maker prior distributions for each post-harvest periods have been stated already; thus, we only pool them together in columns as depicted in Table 3.6 in order to generate the prior probability matrix.

Table 3.6. Post-harvest prior probability matrix


Using matrix algebra, we multiply payoff matrix in Table 3.5 by post-harvest prior probability matrix in Table 3.6. Their product is consecutively multiplied, element by element (Hadamard product) by the mxm identify matrix. (This last operation is done in order to eliminate irrelevant elements.) The result is a mxm diagonal matrix of expected prices under the NONDATA approach, as shown in Equation 3.12.

Where

The expected gain (or loss) per unit sold in period $\mathrm{ph}_{\mathrm{i}}$ is what remains after subtracting from the expected price ep $\left(a_{i}\right)$ its corresponding break-even price bep ${ }_{i}$, that is,

$$
\begin{equation*}
e g\left(a_{i}\right)=e p\left(a_{i i}\right)-\text { bep }_{i} \quad i=1, \ldots m \tag{3.13}
\end{equation*}
$$

The NONDATA Bayesian strategy, as stated before, is to select the course of action $a_{i}$ * such that

$$
\operatorname{eg} *\left(a_{i} *\right)=\max _{i} \operatorname{eg}\left(a_{i}\right)
$$

2. The DATA approach with nondata priors

The computation of the DATA Bayesian strategy seems apparently more complex; that is, because we have to deal with forecast information ( $n$ alternatives) in addition to the actions and the posterior probabilities.

The prediction posterior probability matrix in Table 3.4 relates to a single price forecast interval $\mathrm{z}_{\mathrm{k}}$, thus there is one $\mathbb{P} \mathbb{P} \mathbb{P}$ matrix for each $k$; that is, $\mathbb{P} \mathbb{P} \mathbb{P}_{k}$ $\mathrm{k}=1, \ldots \mathrm{n}$. We multiply payoff matrix in Table 3.5 by each $\mathbb{P} \mathbb{P} \mathbb{P}_{\mathrm{k}}$ matrix; the resulting matrices ( mxm ) are then multiplied, element by element (Hadamard product), by the mxm identify matrix. We come out with a set of mxm diagonal matrices of expected prices, one to each price forecast interval, as shown in Equation 3.14.

$$
\begin{array}{r}
e \mathbb{P}_{k}=\left\|e p\left(a_{i} / z_{k}\right)\right\|=\left\|P_{i j}\right\| \cdot\left\|\mathbb{P} \mathbb{P}_{\operatorname{mxn}} \mathbb{P}_{k}\right\| x\|\underset{\text { mxm }}{\|}\| \partial_{i j} \| \\
k=1, \ldots n
\end{array}
$$

Where


We define the column vector $\mathrm{eg}_{k}$ as the vector of expected gains (or losses) given the $k$ th forecast. Expected gains (or losses) are, as before, the expected prices minus the break-even prices, as shown in Equation 3.15.

$$
\begin{equation*}
e g_{k}=\left\|e p\left(a_{i} / z_{k}\right)-\operatorname{bep}_{i}\right\| \mid \quad k=1, \ldots n \tag{3.15}
\end{equation*}
$$

We define the matrix of expected gains (or losses) for every possible price forecasts as the Matrix $\mathbb{E} G$ whose elements are the $e g_{k}$ column vectors in Equation 3.15, that is:
$\underset{\mathbb{E}}{\mathbb{E}}=\left(e g_{1} \quad e g_{2} \cdot \cdot e g_{k} \cdot \cdot \cdot e g_{n}\right)$

Assuming that $n$ is equal to five $(k=1, \ldots 5)$ the $\mathbb{E} G$ matrix will look as follows:

$\begin{aligned} e g_{i k}= & \text { expected gain from marketing in ph } \\ & \text { cast } Z_{k} \text { for period } i \text { has been issued }\end{aligned}$

A set of mprice predictions $\mathrm{FP} *\left(\mathrm{ph}_{\mathrm{i}}\right)$ is produced prior to the marketing season. We are then able to identify the relevant eg $_{i k}$ * element in each row of matrix $\mathbb{E} G$. The decision problem reduces to choose, among the $m$ elements eg $\left(a_{i}^{*} / Z_{k}^{*}\right)=$ eg $_{i k}{ }^{*}$, the action $a_{i}^{* *}$ whose expected gain is the maximum.

$$
\operatorname{eg} *\left(a_{i}^{* *} / z_{k}\right)=\max _{i} \operatorname{eg}\left(a_{i}^{*} / z_{k}^{*}\right) \quad i=1, \ldots m
$$

$a_{i}^{* *} / Z k_{*}$ is the best post-harvest period the farmer can select to merchandise his grain given that the forecasting model of his choice has predicted the set $F P *\left(p h_{i}\right) \varepsilon Z_{k}{ }^{*}{ }_{i}$ of the marketing season.

## IV. THE PRICE PREDICTION MODELS

In this chapter we develop five price-prediction models; two expectation models were selected because they were known to be used by farmers (9); two linear models were formulated because they appeared to be some of the more logical mechanical models which economists may often use to advise farmers; and one model determined on the basis of the reports issued by an exchange market trading in commodity futures contracts was finally chosen because farmers in Iowa may be well acquainted with it.
A. The Trend Price Model (TPM)

For the Trend Price Model, the difference between prices in the harvest and ith post-harvest months in marketing season t-l is added to the harvest price in marketing season $t$. The resulting value is the predicted price for the ith postharvest month in marketing season $t$. Thus, if the price rises by 20 cents between November and July during the first marketing season (lst year), the price between November and July during the second marketing season (2nd year) would also be expected to rise by 20 cents. This model uses the concept of linear trend in the series and also the relationship between consecutive marketing seasons (Darcovich and Heady, 9). Because of its simplicity, this model may be
used by many corn and soybean producers in the state of Iowa. The model's price predictions in both commodities have been generated for a sample space of 22 marketing seasons (years 1955-1977) using Equation 4.1.

$$
\begin{equation*}
F P\left(p h_{i}\right)_{t}=C P(h)_{t}+\left[C P\left(p h_{i}\right)_{t-1}-C P(h)_{t-1}\right] \tag{4.1}
\end{equation*}
$$

The results of the trend price model (TPM) are shown in Table 4.l.
B. The Moving-Average Price Model (MAPM)

The need for a second expectation model seems apparent. The Moving-Average Price Model is, like the Trend Price Model, one of great simplicity, but still with enough logic to deserve farmers' attention and therefore ours as well. In this model the five-year moving average value of the price series is projected as the predicted value in the sixth year. A five-year period appears to be a convenient length of time over which the memory of many farmers extend. This type of model also has economic applicability; it allows for a flexible rather than a constant trend in a price series. Many farmers believe that last year's price movement is not enough argument to make price predictions for current price movements; thus, they make their predictions in a more conservative and less risky fashion, taking into account the last five years. The moving-average price predictions for

Table 4.1. Past monthly prices as they were predicted by the trendprice expectation model (1955-56 to 1976-77) (\$ per bushel) ${ }^{\text {a }}$

| Marketing Season | CORN |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dec. | Jan. | Feb. | Mar. | Apr. | May | June | July | Aug. | Sept. |
| 1955-1956 | 1.23 | 1.23 | 1.22 | 1.17 | 1.20 | 1.25 | 1.27 | 1.27 | 1.19 | 1.16 |
| 1956-1957 | 1.24 | 1.23 | 1.23 | 1.23 | 1.35 | 1.43 | 1.45 | 1.46 | 1.49 | 1.46 |
| 1957-1958 | 0.91 | 0.90 | 0.85 | 0.85 | 0.89 | 0.91 | 0.92 | 0.91 | 0.89 | 0.78 |
| 1958-1959 | 0.79 | 0.72 | 0.72 | 0.77 | 0.94 | 1.01 | 1.06 | 1.03 | 1.03 | 0.99 |
| 1959-1960 | 1.01 | 1.01 | 1.01 | 1.04 | 1.10 | 1.12 | 1.13 | 1.10 | 1.11 | 1.06 |
| 1960-1961 | 0.68 | 0.69 | 0.67 | 0.69 | 0.77 | 0.81 | 0.88 | 0.89 | 0.86 | 0.85 |
| 1961-1962 | 0.96 | 1.02 | 1.05 | 1.05 | 1.02 | 1.10 | 1.12 | 1.13 | 1.12 | 1.11 |
| 1962-1963 | 0.85 | 0.86 | 0.87 | 0.89 | 0.91 | 0.93 | 0.96 | 0.98 | 0.94 | 0.96 |
| 1963-1964 | 1.04 | 1.10 | 1.11 | 1.11 | 1.12 | 1.16 | 1.23 | 1.26 | 1.25 | 1.28 |
| 1964-1965 | 1.04 | 1.05 | 1.05 | 1.09 | 1.14 | 1.15 | 1.13 | 1.09 | 1.10 | 1.13 |
| 1965-1966 | 1.04 | 1.04 | 1.05 | 1.05 | 1.09 | 1.11 | 1.13 | 1.10 | 1.06 | 1.06 |
| 1966-1967 | 1.27 | 1.32 | 1.32 | 1.28 | 1.34 | 1.38 | 1.39 | 1.46 | 1.52 | 1.53 |
| 1967-1968 | 1.00 | 0.99 | 0.95 | 0.97 | 0.95 | 0.96 | 0.98 | 0.93 | 0.81 | 0.81 |
| 1968-1969 | 1.04 | 1.05 | 1.06 | 1.07 | 1.10 | 1.13 | 1.11 | 1.07 | 1.01 | 1.02 |
| 1969-1970 | 1.06 | 1.10 | 1.10 | 1.09 | 1.11 | 1.19 | 1.18 | 1.18 | 1.17 | 1.13 |
| 1970-1971 | 1.24 | 1.27 | 1.26 | 1.25 | 1.28 | 1.32 | 1.35 | 1.38 | 1.41 | 1.51 |
| 1971-1972 | 1.01 | 1.06 | 1.08 | 1.06 | 1.04 | 1.02 | 1.08 | 1.01 | 0.83 | 0.71 |
| 1972-1973 | 1.25 | 1.24 | 1.24 | 1.25 | 1.28 | 1.30 | 1.29 | 1.30 | 1.29 | 1.36 |
| 1973-1974 | 2.35 | 2.30 | 2.26 | 2.28 | 2.31 | 2.51 | 2.93 | 2.96 | 3.65 | 3.00 |
| 1974-1975 | 3.44 | 3.64 | 3.80 | 3.72 | 3.43 | 3.53 | 3.66 | 4.04 | 4.47 | 4.39 |
| 1975-1976 | 2.27 | 2.04 | 1.85 | 1.66 | 1.68 | 1.70 | 1.68 | 1.73 | 1.97 | 1.79 |
| 1976-1977 | 2.01 | 2.08 | 2.14 | 2.15 | 2.13 | 2.30 | 2.40 | 2.49 | 2.31 | 2.31 |

$a_{\text {The predictions are based on the prices received by farmers in }}$ Iowa as reported by the USDA, ERS.

| SOYBEANS |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Nov. | Dec. | Jan. | Feb. | Mar. | Apr. | May | June | July | Aug. |
| 2.10 | 2.10 | 2.09 | 2.12 | 2.04 | 1.92 | 1.85 | 1.84 | 1.72 | 1.71 |
| 2.10 | 2.12 | 2.22 | 2.26 | 2.38 | 2.64 | 2.99 | 2.90 | 2.44 | 2.40 |
| 2.18 | 2.18 | 2.22 | 2.13 | 2.16 | 2.15 | 2.10 | 2.06 | 2.10 | 2.13 |
| 1.91 | 1.92 | 1.93 | 1.92 | 1.98 | 2.04 | 2.02 | 2.00 | 1.98 | 1.95 |
| 1.93 | 2.01 | 2.02 | 2.03 | 2.05 | 2.07 | 2.09 | 2.06 | 2.03 | 1.95 |
|  |  |  |  |  |  |  |  |  |  |
| 1.94 | 1.89 | 1.91 | 1.90 | 1.90 | 1.93 | 1.89 | 1.87 | 1.86 | 1.94 |
| 2.18 | 2.24 | 2.51 | 2.77 | 2.90 | 3.33 | 3.23 | 2.92 | 2.84 | 2.82 |
| 2.32 | 2.36 | 2.37 | 2.35 | 2.37 | 2.39 | 2.36 | 2.34 | 2.35 | 2.34 |
| 2.57 | 2.62 | 2.66 | 2.72 | 2.73 | 2.68 | 2.72 | 2.73 | 2.69 | 2.67 |
| 2.62 | 2.52 | 2.59 | 2.53 | 2.50 | 2.41 | 2.30 | 2.31 | 2.30 | 2.30 |
|  |  |  |  |  |  |  |  |  |  |
| 2.33 | 2.47 | 2.49 | 2.54 | 2.60 | 2.58 | 2.44 | 2.47 | 2.44 | 2.28 |
| 2.83 | 2.96 | 3.11 | 3.20 | 3.15 | 3.23 | 3.35 | 3.51 | 3.87 | 4.02 |
| 2.49 | 2.52 | 2.44 | 2.35 | 2.38 | 2.34 | 2.32 | 2.34 | 2.30 | 2.19 |
| 2.33 | 2.38 | 2.41 | 2.44 | 2.43 | 2.43 | 2.45 | 2.41 | 2.40 | 2.40 |
| 2.26 | 2.27. | 2.30 | 2.31 | 2.31 | 2.35 | 2.40 | 2.35 | 2.36 | 2.35 |
|  |  |  |  |  |  |  |  |  |  |
| 2.74 | 2.75 | 2.83 | 2.87 | 2.89 | 2.96 | 3.01 | 3.08 | 3.22 | 3.14 |
| 3.03. | 2.92 | 3.03 | 3.09 | 3.08 | 2.96 | 3.02 | 3.18 | 3.40 | 3.30 |
| 2.96 | 3.06 | 3.02 | 3.09 | 3.26 | 3.47 | 3.46 | 3.42 | 3.44 | 3.49 |
| 5.83 | 6.42 | 6.55 | 7.87 | 8.45 | 8.54 | 10.68 | 12.53 | 9.08 | 11.73 |
| 7.78 | 8.30 | 8.50 | 8.70 | 8.55 | 7.75 | 7.85 | 7.79 | 8.79 | 10.22 |
| 4.14 | 3.82 | 2.94 | 2.42 | 1.99 | 2.32 | 1.64 | 1.60 | 1.94 | 2.51 |
| 5.40 | 5.12 | 5.33 | 5.35 | 5.29 | 5.37 | 5.80 | 7.09 | 7.61 | 7.00 |
|  |  |  |  |  |  |  |  |  |  |

corn and soybeans in Iowa have also been generated for a sample space of 22 marketing seasons using Equation 4.2.

$$
\begin{equation*}
F P\left(p h_{i}\right)_{t}=\sum_{y=t-5}^{t-1} C P\left(p h_{i}\right) / 5 \tag{4.2}
\end{equation*}
$$

The subscript $y$ is used as counter of the years ( $t$ ). The results of the moving-average price model (MAPM) are shown in Table 4.2.

## C. The Two-Variable Linear Model (TVLM)

Observable causality implies at least two variables and the simplest relationship between two variables is a linear one. Based on the simplicity of these two abstract concepts many farmers may find attractive the following experimental design.

Assuming that prices for each specific month of the year tend to move along time in a fairly linear fashion, the change in prices may be assumed to be influenced solely by the passing of time in some consistent way, for instance, upwards. If the trend does approximate to a line, the least square method can easily estimate the coefficients of the linear equation. That is, it can be thought that time influences the monthly cash prices as follows:

$$
\begin{equation*}
\mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right)_{\mathrm{t}}=\mathrm{a}+\mathrm{bt}+\mathrm{u} \quad \mathrm{i}=1, \ldots \mathrm{~m} \tag{4.3}
\end{equation*}
$$

Table 4.2. Past monthly prices as they were predicted by the movingaverage price expectation model (1955-56 to 1976-77)
(Price per bushel of the commodity) ${ }^{\text {a }}$

| Marketing Season | CORN |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dec. | Jan. | Feb. | Mar. | Apr. | May | June | July | Aug. | Sept. |
| 1955-1956 |  | 1.450 | 1.434 | 1.434 | 1.460 | 1.488 | 1.496 | 1.496 | 1.484 | 1.488 |
| 1956-1957 | 1.358 | 1.390 | 1.366 | 1.364 | 1.410 | 1.454 | 1.470 | 1.468 | 1.458 | 1.454 |
| 1957-1958 | 1.316 | 1.308 | 1.286 | 1.284 | 1.326 | 1.368 | 1.376 | 1.376 | 1.368 | 1.346 |
| 1958-1959 | 1.204 | 1.186 | 1.176 | 1.176 | 1.250 | 1.300 | 1.324 | 1.316 | 1.306 | 1.268 |
| 1959-1960 | 1.120 | 1.102 | 1.090 | 1.096 | 1.180 | 1.228 | 1.250 | 1.238 | 1.226 | 1.174 |
| 1960-1961 | 1.014 | 0.998 | 0.984 | 1.004 | 1.098 | 1.144 | 1.176 | 1.166 | 1.164 | 1.116 |
| 1961-1962 | 0.938 | 0.936 | 0.928 | 0.948 | 1.012 | 1.058 | 1.088 | 1.080 | 1.070 | 1.026 |
| 1962-1963 | 0.872 | 0.874 | 0.878 | 0.902 | 0.962 | 1.008 | 1.044 | 1.040 | 1.026 | 1.008 |
| 1963-1964 | 0.860 | 0.914 | 0.920 | 0.934 | 0.962 | 1.002 | 1.042 | 1.050 | 1.034 | 1.030 |
| 1964-1965 | 0.894 | 0.924 | 0.930 | 0.946 | 0.972 | 1.010 | 1.044 | 1.050 | 1.034 | 1.046 |
| 1965-1966 | 0.946 | 0.974 | 0.986 | 0.998 | 1.016 | 1.050 | 2.074 | 1.0 .72 | 1.054 | 1.068 |
| 1966-1967 | 0.982 | 1.008 | 1.014 | 1.018 | 1.054 | 1.080 | 1.102 | 1.112 | 1.108 | 1.126 |
| 1967-1968 | 1.056 | 1.080 | 1.076 | 1.080 | 1.108 | 1.132 | 1.152 | 1.148 | 1.128 | 1.142 |
| 1968-1969 | 1.072 | 1.084 | 1.080 | 1.086 | 1.118 | 1.140 | 1.142 | 1.124 | 1.094 | 1.104 |
| 1969-1970 | 1.074 | 1.092 | 1.088 | 1.084 | 1.110 | 1.146 | 1.150 | 1.140 | 1.106 | 1.102 |
| 1970-1971 | 1.064 | 1.088 | 1.080 | 1.074 | 1.098 | 1.138 | 1.144 | 1.146 | 1.126 | 1.142 |
| 1971-1972 | 1.122 | 1.146 | 1.142 | 1.140 | 1.148 | 1.176 | 1.192 | 1.166 | 1.089 | 1.088 |
| 1972-1973 | 1.082 | 1.106 | 1.110 | 1.106 | 1.124 | 1.154 | 1.164 | 1.150 | 1.104 | 1.108 |
| 1973-1974 | 1.154 | 1.166 | 1.160 | 1.158 | 1.176 | 1.240 | 1.338 | 1.338 | 1.442 | 1.314 |
| 1974-1975 | 1.412 | 1.456 | 1.482 | 1.466 | 1.422 | 1.490 | 1.616 | 1.692 | 1.884 | 1.748 |
| 1975-1976 | 1.852 | 1.844 | 1.834 | 1.782 | 1.736 | 1.800 | 1.916 | 1.996 | 2.230 | 2.038 |
| 1976-1977 | 2.050 | 2.046 | 2.044 | 1.998 | 1.952 | 2.054 | 2.178 | 2.290 | 2.524 | 2.356 |
| 1977-1978 | 2.284 |  |  |  |  |  |  |  |  |  |

[^3]SOYBEANS

| Nov. | Dec. | Jan. | Feb. | Mar. | Apr. | May | June | July | Aug. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2.724 | 2.786 | 2.854 | 2.882 | 2.884 | 2.858 | 2.748 | 2.708 |
| 2.532 | 2.578 | 2.588 | 2.614 | 2.698 | 2.778 | 2.848 | 2.844 | 2.664 | 2.630 |
| 2.432 | 2.484 | 2.496 | 2.506 | 2.602 | 2.696 | 2.746 | 2.678 | 2.510 | 2.460 |
| 2.288 | 2.338 | 2.364 | 2.388 | 2.460 | 2.564 | 2.618 | 2.568 | 2.440 | 2.390 |
| 2.158 | 2.180 | 2.210 | 2.206 | 2.236 | 2.278 | 2.324 | 2.286 | 2.168 | 2.142 |
| 2.046 | 2.058 | 2.094 | 2.082 | 2.128 | 2.200 | 2.252 | 2.212 | 2.116 | 2.108 |
| 2.010 | 2.030 | 2.100 | 2.132 | 2.180 | 2.286 | 2.248 | 2.164 | 2.144 | 2.140 |
| 2.016 | 2.044 | 2.108 | 2.154 | 2.200 | 2.312 | 2.278 | 2.198 | 2.172 | 2.160 |
| 2.072 | 2.180 | 2.178 | 2.238 | 2.274 | 2.364 | 2.342 | 2.268 | 2.238 | 2.228 |
| 2.210 | 2.210 | 2.292 | 2.338 | 2.364 | 2.432 | 2.384 | 2.318 | 2.292 | 2.298 |
| 2.324 | 2.362 | 2.444 | 2.502 | 2.540 | 2.598 | 2.530 | 2.474 | 2.444 | 2.402 |
| 2.414 | 2.466 | 2.524 | 2.548 | 2.550 | 2.538 | 2.514 | 2.552 | 2.610 | 2.602 |
| 2.520 | 2.570 | 2.610 | 2.620 | 2.624 | 2.600 | 2.578 | 2.624 | 2.672 | 2.644 |
| 2.554 | 2.604 | 2.642 | 2.646 | 2.646 | 2.632 | 2.606 | 2.642 | 2.696 | 2.672 |
| 2.508 | 2.580 | 2.610 | 2.628 | 2.634 | 2.646 | 2.652 | 2.676 | 2.734 | 2.708 |
| 2.444 | 2.490 | 2.532 | 2.548 | 2.546 | 2.576 | 2.620 | 2.652 | 2.744 | 2.734 |
| 2.536 | 2.534 | 2.568 | 2.578 | 2.584 | 2.574 | 2.606 | 2.638 | 2.702 | 2.642 |
| 2.540 | 2.552 | 2.594 | 2.636 | 2.670 | 2.710 | 2.744 | 2.764 | 2.840 | 2.812 |
| 2.732 | 2.852 | 2.914 | 3.214 | 3.366 | 3.424 | 3.882 | 4.280 | 3.668 | 4.170 |
| 3.272 | 3.490 | 3.586 | 3.924 | 4.046 | 3.936 | 4.404 | 4.800 | 4.386 | 5.176 |
| 4.318 | 4.470 | 4.374 | 4.600 | 4.632 | 4.574 | 4.896 | 5.270 | 4.896 | 5.816 |
| 4.654 | 4.772 | 4.696 | 4.914 | 4.936 | 4.918 | 5.314 | 5.914 | 5.600 | 6.418 |
| 5.298 | 5.492 |  |  |  |  |  |  |  |  |

where $a$ and $b$ are regression coefficients, $u$ is regarded as a random disturbance, and $t$ stands for time in years.

The use of a sample space greater than or equal to ten years is advisable for analysis of long-run price trend. It has the convenience of reducing the relative weight of abnormal years. However, it is unlikely that a farming unit whose price expectations are linear from the past will reckon a price which took place ten years ago. On the other hand, a ten-year linear trend tends to dampen the effects either from recent behavioral changes in structural variables or from new ingredients never observed before. Therefore, we think that the five-year linear trend is better for this particular analysis.

Cash prices are the endogenous variable which represent the result, the final product, after the forces within the system clash at every point in time. The model is extremely simplistic since all forces inside the confines of the economic system, such as supply and demand, are ignored, substituting for their influence on price formation a discrete and monotonically increasing time variable.

Once the least square equation is obtained for each particular month of the marketing season, the mean value of the endogenous variable (price) is predicted for the next $t$ value of the model. This point prediction is the forecasted price for the ith month of the year following the sample
space, that is, $\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)$.
Applying the model continuously along time (i.e., sample space changes from 1 to 5 to 2 to $5+1$, etc.) a sample set of forecasted prices, $F P\left(\mathrm{ph}_{i}\right)$ 's, is generated. The model's price predictions for corn and soybeans for a sample space of 21 marketing seasons are shown in Table 4.3.

The model can easily be represented graphically, assume that the estimated coefficients of the linear equation for the last five Junes have been obtained from corn prices as reported by the USDA (42). Substituting $n+1$ for $t$ in the equation the point prediction $F P$ (June)* is defined as shown in Equation 4.4 and illustrated by Figure 4.1 .

$$
\begin{array}{ll}
\mathrm{FP}(\text { June }) * & =a+b(n+1) \\
3.354 & =1.002+.392(6) \tag{4.4}
\end{array}
$$

D. The Single-Equation Model (SEM)

Any basic book in economics expresses that "the only price that can last, the equilibrium price, is that at which the amount willingly supplied and amount willingly demanded are equal" (Samuelson, 33, p. 63). Equilibrium price is at the intersection point where supply and demand match. Based on these grounds, a model which will be included is the simultaneous supply and demand equations model, that is


Figure 4.1. Two variable linear model. Price prediction for the price of corn in June 1977

Table 4.3. Past monthly prices as they were predicted by the twovariable iinear model ${ }^{\text {a }}$ (1955-56 to 1977-78) ( $\$$ per bushel)

| Marketing <br> Season | Dec. | Jan. | Feb. | Mar. | Apr. | May | June | Juiy | Aug. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1955-1956$ |  | 1.30 | 1.27 | 1.24 | 1.25 | 1.31 | 1.33 | 1.32 | 1.27 |
| $1956-1957$ | 1.23 | 1.15 | 1.17 | 1.14 | 1.23 | 1.31 | 1.33 | 1.34 | 1.34 |
| $1957-1958$ | 1.14 | 1.13 | 1.11 | 1.09 | 1.18 | 1.23 | 1.26 | 1.26 | 1.24 |
| $1958-1959$ | 0.83 | 0.77 | 0.74 | 0.79 | 0.96 | 1.02 | 1.06 | 1.04 | 1.04 |
| $1959-1960$ | 0.77 | 0.74 | 0.73 | 0.80 | 0.92 | 0.95 | 0.97 | 0.93 | 0.96 |
| $1960-1961$ | 0.72 | 0.71 | 0.70 | 0.74 | 0.82 | 0.84 | 0.90 | 0.88 | 0.85 |
| $1961-1962$ | 0.72 | 0.78 | 0.81 | 0.82 | 0.82 | 0.89 | 0.94 | 0.95 | 0.94 |
| $1962-1963$ | 0.84 | 0.91 | 0.93 | 0.93 | 0.87 | 0.91 | 0.93 | 0.97 | 0.93 |
| $1963-1964$ | 0.87 | 0.93 | 0.95 | 0.95 | 0.93 | 0.97 | 1.04 | 1.09 | 1.05 |
| $1964-1965$ | 1.03 | 1.06 | 1.07 | 1.40 | 1.12 | 1.12 | 1.12 | 1.10 | 1.11 |
| $1965-1966$ | 1.14 | 1.13 | 1.13 | 1.14 | 1.21 | 1.21 | 1.22 | 1.17 | 1.15 |
| $1966-1967$ | 1.12 | 1.15 | 1.15 | 1.12 | 1.19 | 1.22 | 1.22 | 1.24 | 1.29 |
| $1967-1968$ | 1.26 | 1.25 | 1.22 | 1.21 | 1.22 | 1.24 | 1.24 | 1.22 | 1.15 |
| $1968-1969$ | 1.11 | 1.12 | 1.10 | 1.10 | 1.10 | 1.13 | 1.13 | 1.11 | 1.01 |
| $1969-1970$ | 1.02 | 1.05 | 1.05 | 1.05 | 1.06 | 1.12 | 1.10 | 1.08 | 1.02 |
| $1970-1971$ | 1.01 | 1.03 | 1.03 | 1.03 | 1.05 | 1.11 | 1.12 | 1.12 | 1.11 |
| $1971-1972$ | 1.17 | 1.24 | 1.26 | 1.23 | 1.24 | 1.25 | 1.31 | 1.29 | 1.21 |
| $1972-1973$ | 1.20 | 1.22 | 1.22 | 1.22 | 1.22 | 1.22 | 1.25 | 1.25 | 1.18 |
| $1973-1974$ | 1.35 | 1.30 | 1.27 | 1.30 | 1.32 | 1.45 | 1.79 | 1.81 | 2.32 |
| $1974-1975$ | 2.19 | 2.30 | 2.41 | 2.37 | 2.14 | 2.31 | 2.61 | 2.92 | 3.62 |
| $1975-1976$ | 3.39 | 3.27 | 3.19 | 3.01 | 2.89 | 3.00 | 3.11 | 3.37 | 3.99 |
| $1976-1977$ | 3.37 | 3.36 | 3.35 | 3.24 | 3.16 | 3.30 | 3.35 | 3.52 | 3.52 |
| $1977-1978$ | 2.80 |  |  |  |  |  |  |  |  |

${ }^{\text {a }}$ Predictions were based on the monthly average price received by Iowar farmers as reported by the USDA.

| SOYBEANS |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Nov. | Jan. | Feb. | Mar. | Apr. | May | June | July |
|  |  | 2.53 | 2.52 | 2.60 | 2.67 | 2.63 | 2.61 | 2.47 |  |
| 2.06 | 2.13 | 2.202 .29 | 2.39 | 2.63 | 2.87 | 2.68 | 2.26 |  |  |
| 2.01 | 1.99 | 2.09 | 2.06 | 2.03 | 2.09 | 2.23 | 2.20 | 2.07 |  |
| 1.85 | 1.77 | 1.82 | 1.72 | 1.72 | 1.69 | 1.71 | 1.69 | 1.64 |  |
| 1.78 | 1.85 | 1.86 | 1.83 | 1.90 | 1.97 | 1.96 | 1.93 | 2.00 |  |
|  |  |  |  |  |  |  |  |  |  |
| 1.89 | 1.88 | 1.86 | 1.85 | 1.82 | 1.76 | 1.60 | 1.61 | 1.76 |  |
| 1.78 | 1.82 | 2.03 | 2.27 | 2.36 | 2.72 | 2.65 | 2.40 | 2.31 |  |
| 2.18 | 2.22 | 2.36 | 2.48 | 2.53 | 2.74 | 2.67 | 2.50 | 2.48 |  |
| 2.37 | 2.41 | 2.50 | 2.58 | 2.62 | 2.66 | 2.66 | 2.61 | 2.58 |  |
| 2.72 | 2.69 | 2.73 | 2.69 | 2.66 | 2.51 | 2.46 | 2.51 | 2.50 |  |
|  |  |  |  |  |  |  |  |  |  |
| 2.82 | 2.86 | 2.81 | 2.73 | 2.71 | 2.47 | 2.35 | 2.51 | 2.50 |  |
| 2.52 | 2.66 | 2.79 | 2.88 | 2.86 | 2.89 | 2.90 | 3.05 | 3.32 |  |
| 2.76 | 2.87 | 2.86 | 2.83 | 2.84 | 2.87 | 2.88 | 2.98 | 3.14 |  |
| 2.52 | 2.63 | 2.61 | 2.63 | 2.63 | 2.67 | 2.76 | 2.76 | 2.81 |  |
| 2.45 | 2.42 | 2.42 | 2.40 | 2.38 | 2.41 | 2.50 | 2.41 | 2.38 |  |
|  |  |  | . |  |  |  |  |  |  |
| 2.25 | 2.21 | 2.25 | 2.26 | 2.29 | 2.34 | 2.36 | 2.32 | 2.30 |  |
| 2.46 | 2.36 | 2.52 | 2.63 | 2.62 | 2.58 | 2.67 | 2.82 | 3.08 |  |
| 2.90 | 2.91 | 2.93 | 3.01 | 3.15 | 3.27 | 3.29 | 3.37 | 3.53 |  |
| 3.52 | 4.01 | 4.10 | 5.19 | 5.74 | 5.87 | 7.56 | 9.07 | 6.34 |  |
| 5.18 | 5.90 | 6.08 | 6.89 | 7.08 | 6.52 | 7.64 | 8.46 | 7.46 |  |
| 7.79 | 7.93 | 7.31 | 7.23 | 6.91 | 6.82 | 6.73 | 6.98 | 6.97 |  |
| 6.85 | 6.47 | 6.24 | 5.88 | 5.46 | 5.43 | 5.25 | 6.08 | 7.20 |  |
| 6.71 | 6.60 |  |  |  |  |  |  |  |  |

$$
\left.\begin{array}{l}
Q\left(\mathrm{ph}_{\mathrm{i}}\right)^{D}=\mathrm{f}(\text { demand variables })+u \\
Q(\mathrm{ph}  \tag{4.5}\\
\mathrm{i}
\end{array}\right)^{\mathrm{S}}=\mathrm{g}(\text { supply variables })+\mathrm{v}
$$

where $u$ and $v$ are regarded as random disturbances, and the variables to be included within the brackets are subject to discussion. The simultaneous equation approach is widely accepted among economists, however not all agree on what to include within the brackets.

There is no doubt that all facets of an economy are interrelated; everything depends upon everything else; yet, most things depend in an essential way upon only a few other things. The fact that each investigator determines, largely based on his experience, what is essential in the analysis precludes agreement about the variables to be included in the supply and demand general model. According to Marshall and his many followers the quantity demanded or the quantity supplied of a product depends upon its price, holding temporarily other things constant; if linearity in the equation is assumed, the relationships in Equation 4.5 can be expressed as follows:

$$
\begin{align*}
& Q\left(\mathrm{ph}_{\mathrm{i}}\right)^{\mathrm{D}}=\mathrm{a}+\mathrm{bCP}\left(\mathrm{ph} \mathrm{i}_{\mathrm{i}}\right)+\mathrm{u}  \tag{4.6}\\
& Q\left(\mathrm{ph}_{\mathrm{i}}\right)^{\mathrm{S}}=\alpha+\beta \mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right)+\mathrm{v}
\end{align*}
$$

This is the simplest case within the family of the two equation simultaneous systems. However, it promises little
econometrically speaking. There is no basis for distinguishing between the two equations since both contain the same variables and both have random disturbances. Thus, the ceteris paribus assumption taken from these has to be relaxed, at least a little. The inclusion of one closely related variable in each equation is a sufficient and necessary condition in order to get out of the statistical trouble. Of course, more than one predetermined variable can be included in the supply and demand equations with the hope to reduce the value of the disturbance terms. Care has to be taken, though, with problems of over- and under-identification, auto-correlation, multi-collinearity, etc. Given that most investigators may agree on the two (perhaps three) leading variables in each equation and disagree in the inclusion of others, the model to be included in this study will be the following:

$$
\begin{align*}
& Q\left(p h_{i}\right)^{D}=a+b C P\left(p h_{i}\right)+c Y_{i}+u  \tag{4,7}\\
& Q\left(p h_{i}\right) S=\alpha+\beta C P\left(p h_{i}\right)+\psi Z_{i}+v
\end{align*}
$$

The quantity of the goods demanded depends on its price and consumer disposable income ( $Y_{i}$ ) while the quantity of the goods supplied depends on its price and on an estimate of the commodity stocks available at the beginning of each $\mathrm{ph}_{i}$, $\left(z_{i}\right)$. The latter is computed by subtracting from total
supply reported (43) at harvest time the cumulative monthly disappearance up to $\mathrm{ph}_{\mathrm{i}}(45)$.

It is to be expected that the variables involved in the demand schedule of consumers, retailers, wholesalers, and producers will not be identical. Thus, the demand for the farm product in question is represented here by a single behavior equation which does not accunately describe the behavior of any subclass of agents (K. A. Fox, 12, p. 25).

Relations 4.7 are the structural form of the model. The process by which the current endogenous variable $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$ is determined, however, may be more clearly seen if we cast the model into its reduced form. By means of successive substitution, the endogenous variable $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$ is expressed as a function only of the predetermined variables $Y_{i}$ and $Z_{i}$, that is

$$
\begin{equation*}
C P\left(\mathrm{ph}_{i}\right)=\left(\frac{\alpha-a}{b-\beta}\right)-\left(\frac{c}{b-\beta}\right) Y_{i}+\left(\frac{\psi}{b-\beta}\right) Z_{i}+\left(\frac{v-u}{b-\beta}\right) \tag{4.8}
\end{equation*}
$$

The coefficients of the reduced form equation were estimated by least squares for a period of 27 years in continuous sets of five consecutive years. Price forecasts FP( $\mathrm{ph}_{\mathrm{i}}$ ) from the model were solved for the year following each set of five years. The sixth-year price forecast is based on an "expected normal" monthly disappearance (the average of percentages observed during the last three years).

The results for corn are shown in Table 4.4. Soybeans were not analyzed by this model due to the lack of monthly disappearance data.
E. The Futures Market (CBT-F)

When the demand and supply forces crash and quantity bought equals the quantity sold the market, it is said, has reached equilibrium and "clearing price" is determined. Along the line between grain producers and consumers many decisions have to be made without perfect knowledge about the clearing market prices. Producers base their production decision on an expected future price; if low they may produce little, if high a lot. Storers base their building and grain investment decisions on what they believe producers will do plus other market expectations peculiar to their line of business. Wholesalers base their actions on what they believe producers, storers, etc., will do on one hand and on what consumers, retailers, etc., will do on the other. Transporters, shippers, processors, all are alike in this regard. Information about others' actions--about supply and demand forces--is indispensable in order to make profitable decisions.

A producer, storer, or processor who faces the risk of price change during growing, storing, or processing season

Table 4.4. Past monthly prices as they were predicted by the single-equation model (1955-56 to 1977-78) (\$ per bushel)

| Marketing Season | CORN |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dec. | Jan. | Feb. | Mar. | Apr . | May | June | July | Aug. | Sept. |
| 1955-1956 | 1.3603 | 1.3029 | 1.2655 | 1.2357 | 1.2643 | 1.3276 | 1.3515 | 1.3225 | 1.2717 | 1.2647 |
| 1956-1957 | 1.2317 | 1.1498 | 1.1648 | 1.1543 | 1.2341 | 1.3147 | 1.3330 | 1.3266 | 1.3354 | 1.3072 |
| 1957-1958 | 1.1457 | 1.1284 | 1.1029 | 1.0877 | 1.1925 | 1.2447 | 1.2706 | 1.2353 | 1.2113 | 1.0864 |
| 1958-1959 | 0.8623 | 0.7755 | 0.7532 | 0.8013 | 0.9656 | 1.0260 | 1.0637 | 1.0400 | 1.0292 | 0.9243 |
| 1959-1960 | 0.7718 | 0.7602 | 0.7685 | 0.8454 | 0.9179 | 0.9442 | 0.9692 | 0.9384 | 0.9675 | 0.8993 |
| 1960-1961 | 0.7116 | 0.7355 | 0.7355 | 0.7817 | 0.8189 | 0.8378 | 0.8967 | 0.8842 | 0.8499 | 0.8285 |
| 1961-1962 | 0.7078 | 0.7852 | 0.8235 | 0.8386 | 0.8227 | 0.8948 | 0.9376 | 0.9545 | 0.9390 | 0.9632 |
| 1962-1963 | 0.8398 | 0.9191 | 0.9392 | 0.9424 | 0.8741 | 0.9083 | 0.9337 | 0.9724 | 0.9276 | 0.9568 |
| 1963-1964 | 0.8620 | 0.9209 | 0.9394 | 0.9442 | 0.9252 | 0.9692 | 1.0358 | 1.0895 | 1.0573 | 1.1125 |
| 1964-1965 | 1.0296 | 1.0490 | 1.0545 | 1.0868 | 1.1151 | 1.1205 | 1.1140 | 1.1062 | 1.1152 | 1.1634 |
| 1965-1966 | 1.1370 | 1.1320 | 1.1274 | 1.1456 | 1.2129 | 1.2074 | 1.2153 | 1.1655 | 1.1442 | 1.1659 |
| 1966-1967 | 1.1126 | 1.1444 | 1.1431 | 1.1067 | 1.1743 | 1.2082 | 1.2050 | 1.2354 | 1.2862 | 1.2891 |
| 1967-1968 | 1.2560 | 1.2550 | 1.2190 | 1.2122 | 1.2196 | 1.2349 | 1.2371 | 1.2210 | 1.1518 | 1.1418 |
| 1968-1969 | 1.1092 | 1.1163 | 1.0969 | 1.0919 | 1.0989 | 1.1276 | 1.1294 | 1.1097 | 1.0154 | 1.0112 |
| 1969-1970 | 1.0247 | 1.0569 | 1.0450 | 1.0469 | 1.0511 | 1.1207 | 1.0975 | 1.0796 | 1.0283 | 1.9965 |
| 1970-1971 | 1.0096 | 1.0366 | 1.0308 | 1.0309 | 1.0516 | 1.1124 | 1.1193 | 1.1158 | 1.1113 | 1.1617 |
| 1971-1972 | 1.1769 | 1.2388 | 1.2594 | 1.2249 | 1.2385 | 1.2518 | 1.3032 | 1.2904 | 1.2134 | 1.1520 |
| 1972-1973 | 1.1825 | 1.2123 | 1.2089 | 1.2033 | 1.2081 | 1.2089 | 1.2426 | 1.2425 | 1.1845 | 1.2090 |
| 1973-1974 | 1.3387 | 1.2978 | 1.2632 | 1.2867 | 1.3101 | 1.4603 | 1.8259 | 1.8366 | 2.4001 | 1.8611 |
| 1974-1975 | 2.1708 | 2.3264 | 2.4307 | 2.3813 | 2.1237 | 2.3066 | 2.6140 | 2.9507 | 3.6320 | 3.2593 |
| 1975-1976 | 3.4035 | 3.3023 | 3.2065 | 3.0239 | 2.8977 | 3.0160 | 3.1383 | 3.4104 | 4.0334 | 3.7386 |
| 1976-1977 | 3.3930 | 3.3505 | 3.3158 | 3.2023 | 3.1641 | 3.3045 | 3.3694 | 3.5265 | 3.5375 | 3.4572 |

may avoid these risks by selling or buying at a known price for future delivery. The price at which the decision maker is willing to contract depends on his expectations, that is, it depends on the quantity and quality of information he owns at each point in time, the weights given to various pieces of information, the experience and judgment he has, etc. The gap between the time to make a market decision and the time to experience its consequences has given rise to more and more active trade on contracts for future delivery. When a contract is agreed between two parties to later buy and sell a commodity, the price specified on it reflects the expectations the buyer and seller had at that time. There is nothing which can prevent these expectations from changing several times from the day of the agreement to the day of delivery. As more or better market information is obtained, price expectations are revised and adjusted while the "price" already agreed on a contract remains fixed. The discrepancy between revised price expectations and the price specified on the contract gives rise to a new market; "the market of the contracts," since contracts acquire value by their own. Trading in futures contracts for specific commodities creates a contracts market not a commodities market. The demand and supply forces of the commodities have not yet been observed, yet the futures contracts trade is done on behalf of them.

A commodity exchange trading in commodity futures contracts is one of the most interesting cases of price formation. Buyers and sellers of futures contracts have a clear position with respect to prices and risk. Those whose main purpose is to get rid of the risk of price variation by transferring it to another are called hedgers. Producers and processors are good examples of hedgers, the majority of them buy or sell in the futures market with the intention to make or accept later delivery of the commodity, thus hedgers hold some position in the market either as sellers or as buyers. On the other hand, there are those who are willing to take the risk of the transaction (risk of price changes) with the hope of making a profit. These are called speculators and buy and sell according to their own price expectations. They have no intention whatsoever to make or take delivery of the physical product, thus they do not hold any specific position in the market. There are similarities between hedgers and speculators which may vary in degree. One thing that is common to any person involved in trading in futures markets is the searching done for price discovery. Every trader has a set of price forecasts of his own which is based on the very best elements he is capable of obtaining. A trader's price predictions are revised as they are confronted by the price predictions of others. The prices at which commodity futures contracts are traded are the result,
therefore, of many experiences and judgements combined, lots of data and information continuously checked and revised. Telser (39, p. 183) shows that a futures price is an average of traders' expectations of the spot price that will prevail at the futures contract's maturity, also he says that for storable commodities, expected futures spot prices and current spot prices differ only by the net marginal cost of storage.

It can be concluded that the "wisdom" on the trading floor of a commodity exchange where commodity futures contracts are heavily traded should be able to predict fairly well the future cash prices. This is the reason why the futures market has been chosen as one of the alternative forecasting methods probably used as reference by many farmers when making their grain marketing decisions. However, its use as reference does not imply any actual or intended trade being done on it.

There are a couple of preliminary steps that should be taken before making use of the futures market as a price forecasting source. First, commodity futures contracts are traded on the pit of most commodity exchanges only for some selected months of the year which implies that, if $\mathrm{ph}_{\mathrm{i}}$ periods are months, either a reduction of the number of alternative courses of action or some sort of interpolation adjustments are needed. Second, the futures prices should be
adjusted back to the cash equivalent in the area where the decision maker is located.

The interpolation process is not going to add basically any further information to the analysis of price formation while it may rather obscure the relevance of each commodity futures price traded on the floor of the exchange. Here, we will only deal with the futures prices traded on the floor of the Chicago Board of Trade. Corn prices traded in November for delivery in the following months of December, March, May, and July serve as the unadjusted price forecasts FP( $\mathrm{ph}_{\mathrm{i}}$ ) $\mathrm{i}=1,4,6,8$. Soybean prices traded in October for delivery in the following months of November, January, March, May, and July serve as the unadjusted price forecasts $F P\left(\mathrm{ph}_{i}\right)$ $i=1,3,5,7,9$.

Cash and futures prices normally tend to move together over time when referred to the same location. It is expected that a cash price will in general validate the futures which predicted it in advance. However, there is normal spread between the cash price and the futures price when both refer to different locations. This spread is related to transportation cost in the first instance, and to disparities in the opportunity cost of capital and differences in the storage cost from one location to the other in the second instance. Usually, this spread is called Basis and it is closely associated only to the transportation cost assuming that the
remaining items are equal to zero. For the sake of generality, we can assume that the Basis (B) includes all three items, thus

$$
\begin{equation*}
\mathrm{B}=\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)_{\mathrm{ch}} .-\mathrm{CP}\left(\mathrm{ph}_{\mathrm{i}}\right)_{\mathrm{I}} . \tag{4.9}
\end{equation*}
$$

where subscripts ch. and I. stand for the locations of the futures market (Chicago) and the decision maker (Iowa) respectively. It was stated in Chapter III that the distributions of the $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$ 's and $\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)$ 's have known distributions with values of central tendency bep ${ }_{i}$ 's, thus

$$
\begin{align*}
& \mathrm{E}\left[\mathrm{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)_{\mathrm{ch}} .-\mathrm{CP}\left(\mathrm{ph} \mathrm{i}_{\mathrm{i}}\right)_{\mathrm{I}}\right]=\overline{\mathrm{F}} \overline{\mathrm{P}}\left(\mathrm{ph} \mathrm{~h}_{\mathrm{i}}\right)_{\mathrm{ch}} .-\overline{\mathrm{C}} \overline{\mathrm{P}}\left(\mathrm{ph} \mathrm{i}_{\mathrm{i}}\right)_{\mathrm{I}} . \\
& =\operatorname{bep}_{i} \mathrm{ch} .-\operatorname{bep}_{\mathrm{i}} \mathrm{I} . \tag{4.10}
\end{align*}
$$

Substituting the break-even prices by their components of Equation 3.1, we have

$$
\begin{equation*}
E[B]=\left[C P(h)_{c h} .-C P(h)_{I} \psi^{i}\right]_{\left(l+r_{c h}\right)^{i}+\Delta S C_{i}} \tag{4.11}
\end{equation*}
$$

where

$$
\begin{aligned}
& \psi=\frac{I+r_{I .}}{I+r_{c h} .} \\
& r_{I}=\text { rate of interest per ph period in Iowa } \\
& r_{c h}=\text { rate of interest per ph period in Chicago } \\
& \Delta S C_{i}=S C_{i} \text { ch. }-S C_{i} I .
\end{aligned}
$$

U.S. Department of Agriculture reports (41) do not show
significant difference in the opportunity cost of capital and storage cost paid (or received) by farmers between Iowa and Illinois. The Basis used here to adjust future prices from the CBT to Central Iowa were obtained from the Cooperative Extension Service at Iowa State University (52) and adjusted to the whole state of Iowa. Future prices were collected from the statistical annual 1975 of the Chicago Board of Trade.

Table 4.5 reports the basis-adjusted future prices for corn and soybean traded in the Chicago Board of Trade for a sample space of 22 years (1955-1977).

Table 4.5. Past monthly prices as they were predicted by the trade in futures ${ }^{\text {a }}$ (contracts in the Chicago Board of Trade) ( $\$$ per bushel)

| Marketing Season | CORN ${ }^{\text {b }}$ |  |  |  | SOYBEANS ${ }^{\text {c }}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dec. | Max. | May | July | Nov. | Jan. | Mar. | May | July |
| 1955-1956 | 0.99 | 1.08 | 1.15 | 1.17 | 2.22 | 2.14 | 2.49 | 2.07 | 1.67 |
| 1956-1957 | 1.13 | 1.18 | 1.28 | 1.31 | 2.06 | 2.23 | 2.43 | 2.18 | 2.11 |
| 1957-1958 | 0.90 | 0.98 | 1.07 | 1.09 | 1.89 | 2.07 | 2.27 | 2.03 | 2.02 |
| 1958-1959 | 0.86 | 0.91 | 1.04 | 1.06 | 1.89 | 1.92 | 2.09 | 1.83 | 1.90 |
| 1959-1960 | 0.82 | 0.91 | 1.03 | 1.04 | 1.92 | 1.98 | 1.99 | 1.98 | 2.03 |
| 1960-1961 | 0.77 | 0.84 | 0.99 | 1.02 | 2.17 | 1.97 | 2.00 | 1.98 | 1.96 |
| 1961-1962 | 0.80 | 0.89 | 0.98 | 1.01 | 2.22 | 2.20 | 2.34 | 2.14 | 2.23 |
| 1962-1963 | 0.84 | 0.84 | 0.95 | 0.98 | 2.61 | 2.30 | 2.39 | 2.19 | 2.26 |
| 1963-1964 | 0.98 | 1.02 | 1.09 | 1.13 | 2.56 | 2.68 | 2.81 | 2.57 | 2.67 |
| 1964-1965 | 1.06 | 1.06 | 1.12 | 1.15 | 2.29 | 2.62 | 2.71 | 2.44 | 2.51 |
| 1965-1966 | 1.14 | 0.96 | 1.09 | 1.11 | 2.78 | 2.30 | 2.51 | 2.31 | 2.31 |
| 1966-1967 | 1.29 | 1.31 | 1.36 | 1.38 | 2.50 | 2.80 | 2.94 | 2.81 | 2.81 |
| 1967-1968 | 0.98 | 1.03 | 1.07 | 1.09 | 2.31 | 2.45 | 2.63 | 2.44 | 2.44 |
| 1968-1969 | 0.98 | 1.03 | 1.07 | 1.05 | 2.25 | 2.33 | 2.44 | 2.34 | 2.34 |
| 1969-1970 | 1.02 | 1.08 | 1.12 | 1.12 | 2.75 | 2.69 | 2.42 | 2.18 | 2.19 |
| 1970-1971 | 1.37 | 1.37 | 1.42 | 1.43 | 2.93 | 2.72 | 2.93 | 2.74 | 2.74 |
| 1971-1972 | 0.97 | 1.02 | 1.06 | 1.05 | 3.20 | 2.93 | 3.05 | 2.93 | 2.94 |
| 1972-1973 | 1.31 | 1.26 | 1.30 | 1.29 | 5.59 | 3.16 | 3.35 | 3.15 | 3.15 |
| 1973-1974 | 2.71 | 2.43 | 2.52 | 2.62 | 8.08 | 5.37 | 5.57 | 5.37 | 5.37 |
| 1974-1975. | 3.31 | 3.61 | 3.64 | 3.36 | 4.65 | 7.92 | 8.70 | 7.89 | 7.89 |
| 1975-1976 | 2.39 | 2.43 | 2.66 | 2.65 | 4.44 | 4.82 | 4.89 | 4.83 | 4.84 |
| 1976-1977 | 2.32 | 2.06 | 2.34 | 2.06 | 5.87 | 5.84 | 5.67 | 5.74 | 5.67 |

$a_{\text {Futures prices }}$ as reported by the Chicago Board of Trade were adjusted to Iowa by subtracting the normal basis between Chicago and Iowa (state average). For the 1955-1965 period the average of price differentials during the last five years was used as the basis. The basis patterns of the Cooperative Extension Service, ISU, were used after 1965.
${ }^{b}$ Corn prices traded in November for delivery in the months listed.
${ }^{C}$ Soybean prices traded in October for delivery in the months listed.

## V. RESULTS

The data used in this chapter are time series of monthly observations for the period 1955-77. The observations refer to the ll-month marketing seasons for corn and soybeans in the state of Iowa. Most of the series are based on official U.S.D.A. reports. However, storage cost figures from 1955 to 1977 were collected from the tariff reports of commercial elevators of the Iowa State Department of Commerce, Warehouse Division. In no cases were the official data adjusted for the effects of specific abnormal years; thus, figures from those years may have played a negative role in some parts of the analysis. There are two reasons for not doing this correction; one is that "good" and "bad" years in agriculture may come at random, thus, some balance may be preserved by leaving all of them. Two, and most importantly, is that there is no general agreement about the border line between "normal" and "abnormal" observations. The entire analysis was carried out for corn and soybeans in Iowa.

In the remainder of this work, the post-harvest periods ( $\mathrm{ph}_{\mathrm{i}}$ ) are identified with months. A weekly analysis (or even daily) is certainly much more meaningful to the farmer; however, serious data problems are faced when this option is adopted.

The marketing seasons of the commodities were defined from a harvest time up to the peak in price prior to the next harvest. The marketing season for corn begins in November and ends in September of the next year. The marketing season for soybeans begins in October, ending in August of the year after. Each season has 11 months; the harvest month and 10 postharvest months. So, eleven courses of action are available. Before turning to the empirical construction of the model and results, we must test the basic assumptions, hypotheses, and features of the model itself.
A. Testing of the Rational-Expectation Hypothesis

The rational-expectation hypothesis has been the cornerstone of the model, its theoretical relevance would be questionable if we did not thoroughly examine the behavior of the break-even prices over a period of years.

Monthly break-even prices were calculated for 22 marketing seasons (1955-56 to 1976-77). Equation 3.1 was used. The corn average cash prices received by farmers in November were used to define the $C P(h)$ values of the marketing seasons. The soybean average cash prices received by farmers in October were used in the same manner. In both cases, the cash prices reported by the Annual Summaries of Agricultural Prices of the U.S.D.A. Economic Research Service were used (42). The
monthly compound interest rates ( $r$ in Equation 3.1) were obtained from the annual rates of interest paid by farmers reported by the Agricultural Finance Statistics (41) (see Table A. 3 in the Appendix). Storage cost figures were collected from the tariff reports of commercial elevators of the Iowa State Department of Commerce (18). The data include carry-in charges, conditioning, and monthly storage rates (average of the 99 counties in Iowa). The three items were clustered in periods going from one month to ten months (see Table A. 4 in the Appendix). The monthly break-even prices obtained from these data are shown in Tables 5.1 and 5.2.

Based on the rational-expectation hypothesis, we stated that the break-even prices are the values of central tendency of the cash prices. The difference between the two sets of prices were calculated for the months of the marketing seasons of the two commodities, that is, the $U_{i_{t}}$ terms of Equation 3.3 were obtained. The mean and variance of every particular month of the marketing seasons were computed (22 observations each sample) in order to accept or reject the hypothesis $u_{i} \bumpeq\left(0, \sigma_{u_{i}}{ }^{2}\right)$ where $\sigma_{u_{i}}{ }^{2}>\sigma_{u_{j}}{ }^{2}$ for $i>j$. The results for both commodities $C$ and S.B. ${ }^{l}$ are reported in Table 5.3. In all cases, we fail to reject the null hypothesis of central tendency. The mean values of the

[^4]Table 5.1. Monthly break-even prices of corn (1954-55 to 1976-77) (\$ per bushel)

| Marketing Season | Dec. | Jan. | Feb. | Max. | Apr. | May |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1954-55 | 1.384493 | 1.404131 | 1.423707 | 1.443057 | 1.462426 | 1.481656 |
| 1955-56 | 1.254843 | 1.274390 | 1.293954 | 1.313091 | 1.332247 | 1.351334 |
| 1956-57 | 1.275807 | 1.295292 | 1.314795 | 1.333903 | 1. 352979 | 1.371951 |
| 1957-58 | 0.984482 | 1.003071 | 1.021622 | 1.039889 | 1.058124 | 1.076310 |
| 1958-59 | 0.925461 | 0.943864 | 0.962036 | 0.979854 | 0.997564 | 1.014824 |
| 1959-60 | 0.961711 | 0.980289 | 0.998696 | 1.016808 | 1.034749 | 1.052743 |
| 1960-61 | 0.808890 | 0.826700 | 0.844523 | 0.862046 | 0.879505 | 0.896852 |
| 1961-62 | 0.913514 | 0.932042 | 0.950588 | 0.968760 | 0.986736 | 1.004498 |
| 1962-63 | 0.893356 | 0.911711 | 0.929826 | 0.947427 | 0.965049 | 0.982351 |
| 1963-64 | 1.009801 | 1.025991 | 1.041903 | 1.057771 | 1.073501 | 1.089250 |
| 1964-65 | 1.038854 | 1.055727 | 1.072220 | 1.088584 | 1.104527 | 1.120448 |
| 1965-66 | 0.992420 | 1.008424 | 1.024337 | 1.040087 | i. 055855 | 1.071555 |
| 1966-67 | 1.239931 | 1.258209 | 1.275886 | 1.292445 | 1.308964 | 1.325433 |
| 1967-68 | 0.990492 | 1.008860 | 1.027248 | 1.043911 | 1.060410 | 1.076817 |
| 1968-69 | 1.036445 | 1.054396 | 1.073192 | 1.089272 | 1.104988 | 1.120728 |
| 1969-70 | 1.081280 | 1.100083 | 1.117661 | 1.134711 | 1.151952 | 1.169217 |
| 1970-71 | 1.286120 | 1.305899 | 1.325706 | 1.344205 | 1.361422 | 1.378668 |
| 1971-72 | 0.981073 | 1.000504 | 1.019126 | 1.035420 | 1.051740 | 1.068085 |
| 1972-73 | 1.203548 | 1.229706 | 1.255894 | 1.277820 | 1.298896 | 1. 319884 |
| 1973-74 | 2.214662 | 2.247570 | 2.280083 | 2.306922 | 2. 334052 | 2.361241 |
| 1974-75 | 3.349558 | 3.390572 | 3.431026 | 3.465792 | 3.500007 | 3.534335 |
| 1975-76 | 2.379144 | 2.419233 | 2.459429 | 2.494708 | 2.529263 | 2.563994 |
| 1976-77 | 2.092833 | 2.140763 | 2.185038 | 2.224031 | 2.263120 | 2.302304 |


| June | July | Aug. | Sept. |
| :---: | :---: | :---: | :---: |
| 1.500904 | 1.520171 | 1.539460 | 1.558768 |
| 1.370437 | 1.389559 | 1.408699 | 1.426856 |
| 1.390941 | 1.409949 | 1.428976 | 1.448021 |
| 1.094512 | 1.112729 | 1.131045 | 1.149206 |
| 1.032098 | 1.049387 | 1.066689 | 1.084006 |
| 1.070751 | 1.088776 | 1.106815 | 1.124870 |
| 0.914212 | 0.931586 | 0.948973 | 0.966373 |
| 1.022275 | 1.040070 | 1.057878 | 1.075704 |
| 0.999669 | 1.017002 | 1.034328 | 1.051672 |
| 1.105017 | 1.120802 | 1.136605 | 1.152429 |
| 1.136389 | 1.152349 | 1.168331 | 1.184332 |
| 1.087274 | 1.103120 | 1.118984 | 1.134869 |
| 1.341926 | 1.358443 | 1.374953 | 1.391488 |
| 1.093243 | 1.109688 | 1.126154 | 1.142640 |
| 1.136487 | 1.152269 | 1.168075 | 1.183901 |
| 1.186504 | 1.203815 | 1.221150 | 1.238510 |
| 1.395945 | 1.413251 | 1.430590 | 1.447957 |
| 1.084452 | 1.100844 | 1.117262 | 1.133703 |
| 1.340903 | 1.361955 | 1.383037 | 1.404150 |
| 2.388496 | 2.415812 | 2.443194 | 2.470638 |
| 3.568784 | 3.603348 | 3.638028 | 3.672830 |
| 2.599255 | 2.634874 | 2.670358 | 2.705954 |
| 2.341588 | 2.380966 | 2.420444 | 2.460020 |

Table 5.2. Monthly break-even prices of soybeans (1954-55 to 1976-77) (\$ per bushel)

| Marketing <br> Season | Nov. | Dec. | Jan. | Feb. | Mar. | Apr. |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1954-55$ | 2.548940 | 2.573044 | 2.597103 | 2.620952 | 2.644838 | 2.668602 |  |
|  |  |  |  |  |  |  |  |
| $1955-56$ | 2.118139 | 2.140998 | 2.163885 | 2.186357 | 2.208861 | 2.231310 |  |
| $1956-57$ | 2.149143 | 2.171978 | 2.194842 | 2.217326 | 2.239788 | 2.262161 |  |
| $1957-58$ | 2.068709 | 2.091542 | 2.114354 | 2.136899 | 2.159429 | 2.181926 |  |
| $1958-59$ | 1.989693 | 2.012346 | 2.034787 | 2.056890 | 2.078903 | 2.100481 |  |
| $1959-60$ | 1.975824 | 1.988533 | 2.021090 | 2.043365 | 2.065490 | 2.087684 |  |
|  |  |  |  |  |  |  |  |
| $1960-61$ | 1.943595 | 1.966135 | 1.988704 | 2.010995 | 2.033240 | 2.055395 |  |
| $1961-62$ | 2.219015 | 2.243070 | 2.267165 | 2.290912 | 2.314485 | 2.337867 |  |
| $1962-63$ | 2.249177 | 2.273379 | 2.297366 | 2.320866 | 2.344411 .2 .367662 |  |  |
| $1963-64$ | 2.546399 | 2.569217 | 2.591784 | 2.614336 | 2.636779 | 2.659270 |  |
| $1964-65$ | 2.535397 | 2.558844 | 2.581938 | 2.604934 | 2.627537 | 2.650148 |  |
|  |  |  |  |  |  |  |  |
| $1965-66$ | 2.328259 | 2.350131 | 2.371938 | 2.393607 | 2.415320 | 2.436992 |  |
| $1966-67$ | 2.816826 | 2.842030 | 2.866663 | 2.890210 | 2.913746 | 2.937265 |  |
| $1967-68$ | 2.487271 | 2.512450 | 2.537683 | 2.561217 | 2.584624 | 2.607967 |  |
| $1968-69$ | 2.372602 | 2.396739 | 2.421750 | 2.444075 | 2.466064 | 2.488107 |  |
| $1969-70$ | 2.236695 | 2.260940 | 2.283984 | 2.306527 | 2.329287 | 2.352097 |  |
| $1970-71$ | 2.765020 | 2.791609 | 2.818263 | 2.844981 | 2.870513 | 2.895945 |  |
| $1971-72$ | 2.991672 | 3.021754 | 3.051079 | 3.078576 | 3.106150 | 3.133801 |  |
| $1972-73$ | 3.145090 | 3.185113 | 3.225220 | 3.260237 | 3.294276 | 3.328281 |  |
| $1973-74$ | 5.597459 | 5.655081 | 5.712855 | 5.761828 | 5.810509 | 5.859341 |  |
| $1974-75$ | 8.306628 | 8.379554 | 8.452086 | 8.518176 | 8.584555 | 8.650881 |  |
| $1975-76$ | 4.985091 | 5.045417 | 5.105971 | 5.160461 | 5.215184 | 5.270041 |  |
| $1976-77$ | 5.916208 | 5.997695 | 6.074451 | 6.143580 | 6.212985 | 6.282667 |  |
|  |  |  |  |  |  |  |  |


|  |  |  |  |
| :--- | :---: | :---: | :--- |
| May | June | July | Aug. |
| 2.692401 | 2.716237 | 2.740113 | 2.764025 |
|  |  |  |  |
| 2.253788 | 2.276298 | 2.298838 | 2.321409 |
| 2.284564 | 2.307000 | 2.329466 | 2.351964 |
| 2.204455 | 2.227018 | 2.249796 | 2.272236 |
| 2.122092 | 2.143735 | 2.165407 | 2.187115 |
| 2.109908 | 2.132167 | 2.154457 | 2.176781 |
|  |  |  |  |
| 2.077583 | 2.099805 | 2.122059 | 2.144345 |
| 2.361289 | 2.384753 | 2.408254 | 2.431796 |
| 2.390955 | 2.414289 | 2.437641 | 2.461038 |
| 2.681809 | 2.704396 | 2.727028 | 2.749710 |
| 2.672807 | 2.695516 | 2.718276 | 2.741084 |
|  |  |  |  |
| 2.458707 | 2.480577 | 2.502492 | 2.524452 |
| 2.960836 | 2.984465 | 3.008117 | 3.031825 |
| 2.631362 | 2.654807 | 2.678304 | 2.701853 |
| 2.510197 | 2.532340 | 2.554535 | 2.576780 |
| 2.374954 | 2.397861 | 2.420819 | 2.443830 |
|  |  |  |  |
| 2.921442 | 2.947005 | 2.972637 | 2.998335 |
| 3.161526 | 3.189329 | 3.217208 | 3.245162 |
| 3.362398 | 3.396600 | 3.430887 | 3.465258 |
| 5.908340 | 5.957498 | 6.006822 | 6.056309 |
| 8.717512 | 8.784431 | 8.851641 | 8.919154 |
| 5.325550 | 5.381292 | 5.437277 | 5.493499 |
| 6.352631 | 6.422872 | 6.493399 | 6.564210 |

Table 5.3. Test of the central tendency hypothesis and the assumption of increasing variance over the marketing season

| Months of the marketing season (phí) |  |  | $s^{2} u_{i}$ <br> (Variance) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Corn | Soybeans | Corn | Soybeans |
| November | - | $\begin{gathered} -0.0702 \\ (-1.333) \end{gathered}$ | - | 0.061 |
| December | $\begin{gathered} -0.0006 \\ (-0.039) \end{gathered}$ | $\begin{gathered} -0.0375 \\ (-0.445) \end{gathered}$ | 0.006 | 0.156 |
| January | $\begin{gathered} -0.0194 \\ (-0.733) \end{gathered}$ | $\begin{aligned} & -0.0705 \\ & (-0.606) \end{aligned}$ | 0.015 | 0.297 |
| February | $\begin{gathered} -0.0429 \\ (-1.143) \end{gathered}$ | $\begin{gathered} -0.0311 \\ (-0.181) \end{gathered}$ | 0.031 | 0.645 |
| March | $\begin{gathered} -0.0703 \\ (-1.646) \end{gathered}$ | $\begin{array}{r} -0.0397 \\ (-0.193) \end{array}$ | 0.040 | 0.927 |
| April | $\begin{aligned} & -0.0688 \\ & (-1.735) \end{aligned}$ | $\begin{aligned} & -0.0465 \\ & (-0.225) \end{aligned}$ | 0.035 | 0.938 |
| May | $\begin{gathered} -0.0365 \\ (-0.849) \end{gathered}$ | $\begin{gathered} 0.0212 \\ (0.071) \end{gathered}$ | 0.041 | 1.942 |
| June | $\begin{gathered} -0.0100 \\ (-0.188) \end{gathered}$ | $\begin{array}{r} 0.1215 \\ (0.330) \end{array}$ | 0.062 | 2.978 |
| July | $\begin{array}{r} -0.0095 \\ (-0.162) \end{array}$ | $\begin{gathered} 0.0185 \\ (0.077) \end{gathered}$ | 0.076 | 1.278 |
| August | $\begin{array}{r} 0.0030 \\ (0.037) \end{array}$ | $\begin{array}{r} 0.1574 \\ (0.488) \end{array}$ | 0.157 | 2.283 |
| September | $\begin{gathered} -0.0654 \\ (-0.914) \end{gathered}$ |  | 0.113 | - |
| Null Hypothesis <br> 95\% Confidence interval | $\begin{aligned} & \text { Но: } \overline{\mathrm{U}}_{\mathrm{i}} \\ & \overline{\mathrm{U}}_{\mathrm{i}} \pm 2 . \end{aligned}$ | $0.000$ |  | $>\sigma_{u_{j}}^{2}$ |

differences are not significantly different from zero, the statistical test confirms that bep ${ }_{i}$ prices do represent the values of central tendency of the cash prices $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$ for the ten months of the marketing seasons of the products $C$ and S.B. Also, the variances of the commodities show an increasing trend over the months of the marketing season. The assumption of increasing variance stated in•Equation 3.3 seems to be consistent with the results of Table 5.3.

Yet, we still must establish the kind of distribution we are dealing with. Based on the central limit theorem, we assumed in Chapter III that the distribution of cash prices around their bep $\mathrm{p}_{\mathrm{i}}$ parameters was likely to be normal. The assumption was largely due to the fact that future demands for grain are not accurately anticipated relative to supplies, generating some kind of random distribution. The tests of skewness and kurtosis were applied to the data in order to determine the degree of validity of the assumption of normality.

Skewness measures the degree of deviation from symmetry; therefore, it is the statistic we need to determine the degree to which $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$-bep $\mathrm{p}_{\mathrm{i}}$ distributions approximate a normal curve. The Pearsonian coefficient of skewness is accurate enough if the number of observations exceeds at least 150 (36, pp. 85-86), however, when used in small samples it does not aiways measure what it is supposed to. Mood,

Graybill, and Boes (27, p. 76) provide a measure of skewness $S=($ mean-median) $/($ standard deviation) which is to be applied to samples with few observations. The measure of skewness takes a value of zero when the distribution is completely symmetric bell-shaped curve, and it can be proved that $-1 \leq S \leq 1$. The coefficient was calculated for the ten $U_{i}$ distributions of the commodities (samples of 22 observations). The results are reported in Table 5.4.

Kurtosis is a measure of the relative peakedness or flatness of the curve defined by the distribution of cases. For sample sizes less than 200, no tables of the significance levels of the coefficient of kurtosis (fourth moment of the sample about its mean) are at present available (36, p. 88). R. C. Geary (13) developed an alternative test criterion for kurtosis. $\underline{a}=(m e a n$ deviation)/(standard deviation). The expected value of $a$ when computed for a sample of 21 observations from a normal distribution is 0.80792 . Leptokurtic distributions (peaked) produce lower values of $\underline{a}$, and platykurtic distributions (flat) produce higher values of $\underline{a}$ (13 and 22, p. 28). The coefficient a was calculated for the $\mathrm{U}_{\mathrm{i}}$ distributions of the commodities, the results are shown in Table 5.4.

In most cases the coefficient of skewness is not considerably different from zero. Fifty-five percent of the cases depart from perfect normality by less than 0.10 ,

Table 5.4. Test of skewness and kurtosis of the $U_{i}$ $\left[\mathrm{U}_{\mathrm{i}}=\mathrm{CP}\left(\mathrm{ph} \mathrm{h}_{\mathrm{i}}\right)-\mathrm{bep}_{\mathrm{i}}\right]$ distributions of the months of the marketing season

| Months of <br> the <br> marketing season | Skewness (S) |  | Kurtosis (a) |  |  |
| :--- | ---: | ---: | :--- | :--- | :--- |
| Corn | - | -0.2789 | - | 0.6402 |  |
| November | -0.0753 | -0.0461 | 0.7829 | 0.5195 |  |
| December | -0.0964 | -0.1536 | 0.6697 | 0.4861 |  |
| January | -0.1325 | -0.0378 | 0.6055 | 0.4509 |  |
| February | -0.1932 | -0.0330 | 0.5707 | 0.4339 |  |
| March | -0.2209 | -0.0208 | 0.5266 | 0.5099 |  |
| April | -0.3181 | 0.0520 | 0.5490 | 0.4593 |  |
| May | -0.0582 | 0.1040 | 0.5062 | 0.4524 |  |
| June | 0.0289 | 0.0707 | 0.5612 | 0.5132 |  |
| July | 0.1335 | 0.1746 | 0.5744 | 0.5118 |  |
| August | 0.0556 | - | 0.6252 | - |  |
| September |  |  |  |  |  |

Null Hypothesis Ho: $S=0.0$
Ho: $a=.80792$
Tolerance
$-1 \leq 5 \leq 1$
Upper 1\% $=0.9001$
Lower 1\% $=0.6950$
eighty-five percent by less than 0.20. In other words, only fifteen percent of the coefficients has a level which can be described as "moderate" (22). Taking into account that none of the means of central tendency of the $C P\left(\mathrm{ph}_{i}\right)$ - bep ${ }_{i}$ distributions was significantly different from zero, we fail to recognize skewness in the sample distributions. In contrast to these results, the coefficient of kurtosis does show $\mathfrak{a}$ to be significantly different from its expected mean ( 0.807792 ) in all cases but one. The distributions of the samples show that density is more peaked around its center than the density of a normal curve (leptokurtic). We mentioned at the beginning of this chapter that the observations from abnormal years were not excluded from the samples, so that our small samples are likely to contain a number of atypical cases located (at random) at either extreme of the distributions which have a disproportionate effect on the shape of the distributions. This case would have been revealed in the statistical analysis by a relative peakedness of the curve defined by the distribution of all cases (typical and atypical). Yet, we think that this is only a possible explanation of the results. On the other hand, by ignoring leptokurtosis and assuming normality of the distributions we overestimate the degree of dispersion of the data. Overestimation of the variance expands the feasible range of the outcomes (cash prices), letting quasi-unusual
observations fall inside the confidence boundaries of the distribution (say 95 percent C.I.). To clarify this point, we elaborate a little bit more on it below.

Towards the construction of the Bayesian Model, the confidence boundaries (feasible range) of the cash prices were defined. The variance and the standard deviation of the $\mathrm{U}_{\bar{i}_{t}}$ samples were computed in successive sets of five years, and extrapolated to the sixth. Tables of the standard normal distributions were used to define the boundaries at 95 percent confidence intervals from equations of the following form;

$$
\begin{equation*}
\mathrm{BP}_{\text {cit }}=\text { bep }_{\text {it }} \pm \mathrm{Z}_{(.95)} \sigma_{\text {uit }} \tag{5.1}
\end{equation*}
$$

where ${ }^{B P}{ }_{\text {cit }}$ are the points on the confidence price boundary. This procedure was carried out for the ten months of the marketing season over a period of 22 years. In all cases, the boundaries were defined exclusively with data available only up to the beginning of the marketing season, thus, the feasible range is always defined prior to the observation of the outcome. The resulting sets of the feasible range are depicted in Figures 5.1 and 5.2. The cash prices (actual outcomes) are also reproduced on the graphs.

The diagrams amply confirm the results of the statistical analysis. Ninety-six percent of the observations fall inside the feasible range of the outcome. Looking closer to the

Figure 5.1. Feasible range of corn cash prices for the months of the marketing seasons from 1955-56 to 1976-77

Upper bound (•)
Lower bound (*)
Cash prices (+)











Figure 5.2. Feasible range of soybeans cash prices for the months of the marketing seasons from 1955-56 to 1976-77

Upper bound (.)
Lower bound (*)
Cash prices (+)










data, we find that fifteen of the twenty observations falling outside the range belong to the year of 1973 , perhaps the upmost abnormal year of all. It is to expect that other years (less than 1973 but still abnormal) would have come out of the range if the "normal" standard deviation would have been corrected from the results of kurtosis.
B. Testing of the Price Forecasting Models

Five price prediction models were presented in Chapter IV. Monthly price predictions were computed from them over a period of 26 years (Tables 4.1, 4.2, 4.3, 4.4 and 4.5). It was stated in Chapter III that any meaningful forecasting model must make predictions in the range of feasibility of the event which is predicted; predictions outside the range have no theoretical support. They are irrelevant alternatives from the probabilistic point of view. Predictions constrained to a subset of the feasible range of the outcome fail to predict outcomes that do actually occur. Based on these assumptions, our first hypothesis was $\operatorname{FP}\left(\mathrm{ph}_{\mathrm{i}}\right)_{t}=\operatorname{bep}_{\mathrm{i}_{t}}+\varepsilon_{\mathrm{i}_{t}}$ (Equation 3.5), where the biasness factor $\bar{\varepsilon}_{i}$ (mean value of the discrepancies over a sample of years) must be equal to zero. Our second hypothesis was that the variance of $F P\left(\mathrm{ph}_{\mathrm{i}}\right)$ and bep $_{i}$ distributions must be equal, since it has been proven that $C P\left(\mathrm{ph}_{\mathrm{i}}\right) \sim\left(\mathrm{bep}_{i}, \sigma_{b e p_{i}}^{2}\right)$. The results of testing
both hypotheses are shown in Tables 5.5 and 5.6. The student's t-test and the $F$-test of the equality of two variances (36, pp. ll6-119) were employed.

Models TPM, TVLM, SEM, and CBT-F are unbiased with range of prediction equal to the feasible range of the cash prices. For these four models, the biasness factor $\bar{\varepsilon}_{i}$ is not significantly different from zero (95 percent C.I.) for all the months of the marketing season of corn and soybeans. The F-test of the equality of two variances applied to the data shows that; the variances of the $\mathrm{FP}\left(\mathrm{ph}_{i}\right)$ distributions of the four models are in no case different from the variances of the bep ${ }_{i}$ distributions at 95 percent level of significance. In most cases, the MAPM model fails to pass the tests of central tendency and equal variance. The MAPM model fails the t-test in five months for corn and eight months for soybeans at the 95 percent level of significance; that is, we reject the null hypothesis $\bar{\varepsilon}_{i}=0$ in 65 percent of the cases. The feasible range of the forecast defined by this model is significantly smaller than the feasible range of the cash prices in all cases. We reject the null hypothesis of equal variance $s_{F P\left(p h_{i}\right)}^{2}=s_{b e p_{i}}^{2}$ for all the

[^5]Table 5.5. Tests of central tendency and variance of the price prediction models (corn)


Table 5.6. Tests of central tendency and variance of the price prediction models (soybeans)

| Months of the marketing season ( $\mathrm{ph}_{\mathrm{i}}$ ) | $\qquad$ <br> $\bar{\varepsilon}$ - Biasness Factor (Student's t) |  |  |  | $\begin{gathered} s^{2}{ }_{F P} / s^{2} \text { bep } \\ (\mathrm{F}-\text { test) } \end{gathered}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\ldots$ TPM | MAPM | TVLM | CBT-F | TPM | MAPM | TVLM | CBT-F |
| November | $\begin{array}{r} 0.0924 \\ (0.277) \end{array}$ | $\begin{array}{r} -0.3344 \\ (-1.769) \end{array}$ | $\begin{gathered} 0.0968 \\ (0.257) \end{gathered}$ | $\begin{gathered} 0.0883 \\ (0.259) \end{gathered}$ | $1.0480{ }^{\text {a }}$ | $3.2635^{\text {a }}$ | 1.2302 | $1.0012^{\text {a }}$ |
| December | $\begin{array}{r} 0.0919 \\ (0.263) \end{array}$ | $\begin{gathered} -0.3039 \\ (-1.524) \end{gathered}$ | $\begin{aligned} & 0.1206 \\ & (0.315) \end{aligned}$ | - | 1.0260 | $2.9955^{\text {a }}$ | 1.2327 | - |
| January | $\begin{gathered} -0.0538 \\ (-0.155) \end{gathered}$ | $\begin{array}{r} -0.4275 \\ (-2.949) \end{array}$ | $\begin{gathered} -0.0787 \\ (-0.2431) \end{gathered}$ | $\begin{array}{r} -0.0552 \\ (-0.169) \end{array}$ | $1.0036{ }^{\text {a }}$ | $5.7759^{\text {a }}$ | $1.1579{ }^{\text {a }}$ | $1.1349{ }^{\text {a }}$ |
| February | $\begin{gathered} -0.0162 \\ (-0.042) \end{gathered}$ | $\begin{array}{r} -0.3870 \\ (-2.397) \end{array}$ | $\begin{aligned} & -0.0212 \\ & (-0.062) \end{aligned}$ | - | 1.2151 | $4.7376^{\text {a }}$ | $1.0556^{\text {a }}$ | - |
| March | $\begin{gathered} -0.0266 \\ (-0.067) \end{gathered}$ | $\begin{array}{r} -0.3730 \\ (-2.288) \end{array}$ | $\begin{array}{r} -0.0302 \\ (-0.089) \end{array}$ | 0.0293 | 1.2681 | $4.7262^{\text {a }}$ | $1.0958^{\text {a }}$ | $1.0575^{\text {a }}$ |
| April | $\begin{gathered} -0.0352 \\ (-0.093) \end{gathered}$ | $\begin{array}{r} -0.3689 \\ (-2.378) \end{array}$ | $\begin{array}{r} -0.0469 \\ (-0.146) \end{array}$ | - | 1.1211 | $5.3031{ }^{\text {a }}$ | $1.2333^{\text {a }}$ | - |
| May | $\begin{array}{r} 0.0306 \\ (0.066) \end{array}$ | $\begin{array}{r} -0.3194 \\ (-1.716) \end{array}$ | $\begin{array}{r} 0.0467 \\ (0.123) \end{array}$ | $\begin{aligned} & -0.2344 \\ & (-0.711) \end{aligned}$ | 1.6414 | $3.7512^{\text {a }}$ | 1.1157 | $1.1949{ }^{\text {a }}$ |
| June | $\begin{array}{r} 0.1292 \\ (0.237) \end{array}$ | $\begin{array}{r} -0.2854 \\ (-1.266) \end{array}$ | $\begin{gathered} 0.1632 \\ (0.358) \end{gathered}$ | - | 2.2489 | $2.5955^{\text {a }}$ | 1.5699 | - |
| July | $\begin{gathered} 0.0243 \\ (0.052) \end{gathered}$ | $\begin{array}{r} -0.4155 \\ (-2.102) \end{array}$ | $\begin{array}{r} 0.0231 \\ (0.059) \end{array}$ | $\begin{aligned} & -0.2938 \\ & (-0.893) \end{aligned}$ | 1.6164 | $3.4373^{\text {a }}$ | 1.1444 | $1.2411^{\text {a }}$ |
| August | $\begin{array}{r} 0.1613 \\ (0.283) \end{array}$ | $\begin{array}{r} -0.3287 \\ (-1.254) \end{array}$ | - | - | 2.3724 | $1.9890^{\text {a }}$ | - | - |
| Null <br> Hypothesis Tolerance | (2.07 | Ho: <br> ${ }^{4)} \alpha=.05$ | $\begin{gathered} \bar{E}_{i}=0 \\ (2.780)_{\alpha=} \end{gathered}$ | . 01 |  | Ho: <br> 15) $=2$ | $\begin{aligned} & 2 / s^{2}= \\ & .04 F_{( } . \end{aligned}$ | $=2.78$ | $\mathrm{a}_{\mathrm{s}_{\mathrm{bep}}} / \mathrm{s}_{\mathrm{FP}}{ }^{2}$

months of the M.S. ${ }^{1}$ of corn and for nine of the ten months of the M.S. of S.B. The usefulness of the MAPM model as a price forecasting method for C and S.B. in Iowa is very limited; however, we do not exclude it from further consideration simply for analytical reasons.
C. Defining the Empirical Elements of the Bayesian Model

Payoff Matrix: The feasible range of the outcome for the months of the M.S. was divided into five discrete intervals, all equally likely, mutually exclusive, and collectively exhaustive of the range. The five states of nature of section 3.4 (same as the five price forecast intervals of section 3.5) were determined beginning from the upper bound of the feasible range. The states of nature (forecast intervals) so defined were calculated over the period of 22 years (M.S.'s 1955-56 to 1976-77).

The mid-range values of the five intervals were used as their representative expected prices. In other words, midrange values of the intervals are the prices of the payoff matrices ( $P_{i j}$ values in Figure 3.3). The payoff matrix for 1977 is shown in Table 5.7 (1975 and 1976 payoff matrices are shown in the Appendix).

The Conditional Probabilities: The monthly cash prices of 22 years (1955-1977) were traced over the five

[^6]Table 5.7. Payoff matrices of corn and soybeans for the 1976-77 marketing season (\$ per bushel)

|  | $\theta_{1 i}$ | $\theta_{2 i}$ | $\theta_{3 i}$ | $\theta_{4 i}$ | $\theta_{5 i}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $a_{1}$ (Dec.) | 3.3771 | 2.4575 | 2.0928 | 1.7281 | 0.8079 |  |
| $a_{2}$ (Jan.) | 3.7523 | 2.5982 | 2.1407 | 1.6833 | 0.5292 |  |
| $a_{3}$ (Feb.) | 3.7815 | 2.6382 | 2.1850 | 1.7319 | 0.5885 |  |
| $a_{4}$ (Mar.) | 3.6986 | 2.6426 | 2.2240 | 1.8054 | 0.7494 |  |
| $a_{5}$ (Apr.) | 3.6207 | 2.6485 | 2.2631 | 1.8777 | 0.9055 |  |
| $a_{6}$ (May) | 3.6574 | 2.6869 | 2.3023 | 1.9176 | 0.9472 |  |
| $a_{7}$ (June) | 3.6487 | 2.1126 | 2.3416 | 1.9705 | 1.0344 |  |
| $a_{8}$ (July) | 3.8424 | 2.7958 | 2.3810 | 1.9661 | 0.9195 |  |
| $a_{9}$ (Aug.) | 4.0606 | 2.8858 | 2.4204 | 1.9550 | 0.7808 |  |
| $a_{10}$ (Sep.) | 4.0071 | 2.8992 | 2.4600 | 2.0209 | 0.9129 |  |
|  |  |  |  | SOYBEANS |  |  |
| $a_{1}$ (Nov.) | 8.8787 | 6.7571 | 5.9162 | 5.0753 | 2.9538 |  |
| $a_{2}$ (Dec.) | 8.6647 | 6.7548 | 5.9971 | 5.2407 | 3.3307 |  |
| $a_{3}$ (Jan.) | 8.6634 | 6.8094 | 6.0745 | 5.3396 | 3.4855 |  |
| $a_{4}$ (Feb.) | 8.5214 | 6.8186 | 6.1436 | 5.4686 | 3.7656 |  |
| $a_{5}$ (Mar.) | 8.5028 | 6.8629 | 6.2129 | 5.5630 | 3.9232 |  |
| $a_{6}$ (Apr.) | 8.3467 | 6.8685 | 6.2826 | 5.6967 | 4.2185 |  |
| $a_{7}$ (May) | 9.7960 | 7.3301 | 6.3527 | 5.3753 | 2.9094 |  |
| $a_{8}$ (June) | 11.3281 | 7.8153 | 6.4229 | 5.0305 | 1.5176 |  |
| $a_{9}$ (July) | 9.1835 | 7.2570 | 6.4934 | 5.7298 | 3.8034 |  |
| $a_{10}$ (Aug.) | 10.7798 | 7.7608 | 6.5642 | 5.9676 | 2.3487 |  |

states of nature. The monthly price predictions of the forecasting models were also traced in the same manner for the same period. The frequency distribution of the events (cash prices and price predictions) over the intervals (states of nature or forecast intervals) gave rise to the elements of the conditional probability function. Tables of the monthly conditional probabilities were made in arrays of $5 \times 5$ (see conditional probability tables in the Appendix).

The Prior Probabilities: The monthly nondata prior probabilities over the states of nature were not obtained bysampling the farmer's expectations about the prices reported in the payoff matrix for the ten months of the marketing season (meaningful samples can only be made at harvest time, before the cash prices of the particular M.S. are observed). Instead, four plausible sets of nondata priors were formulated on the belief that they represent roughly the infinite number of possibilities.

The first nondata prior probability vector is based on the assumption that the farmer thinks that all five prices reported by the payoff matrix for a particular month are equally likely. This does not mean that the farmer is indifferent about the five prices of the payoff matrix. Obviously, he prefers a higher price to occur than a lower; however, he still thinks that both have the same chances to occur. We call this nondata prior probability vector
"I", its values are shown in Table 5.8.

Table 5.8. Hypothetical nondata prior probability vectors with respect to the monthly prices reported by the payoff matrix

| States of nature | Prior Probabilities |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\overline{\mathrm{I}}$ | 0 | N | $\overline{\mathrm{P}}$ |
| $\theta_{1}$ | . 20 | . 30 | . 10 | . 05 |
| $\theta_{2}$ | . 20 | . 35 | . 20 | . 10 |
| $\theta_{3}$ | . 20 | . 20 | . 40 | . 20 |
| ${ }^{4} 4$ | . 20 | . 10 | . 20 | . 35 |
| $\theta_{5}$ | . 20 | . 05 | . 10 | . 30 |

$a_{\text {Represented }}$ by specific monthly sets of prices of the payoff matrix.

The second nondata prior probability vector is based on the assumption that the farmer's expectations are such that the higher prices receive higher probabilities to be observed and lower prices receive lower probability values. We call this the nondata prior probability vector "O" and it is shown in Table 5.8.

The third nondata prior probability vector assumes that the farmer has a strong appeal for the central value of the five prices but still he remains aware of the feasibility of the others. We call this prior probability vector "N", it is also shown in Table 5.8.

The last nondata prior probability vector is based on the assumption that the farmer is pessimistic about the higher prices reported on the payoff matrix (for the month in question). Thus, he gives higher probabilities to lower prices and lower probabilities to higher prices. We call this prior probability vector "P" with values as shown in Table 5.8.

The data-priors of the five forecasting models were obtained by tracing the monthly forecasts (month by month) over the five states of nature for a period of 22 years (19551977). The relative frequency distributions of the forecasts over the states of nature gave rise to the monthly data priors of the forecasting models. In contrast to the conditional probabilities, the data priors make no use of the distribution of cash prices over the states of nature, monthly conditionals are two classification probability tables (states of nature $x$ forecasts) while monthly data priors are one classification probability vectors (states of nature). For each forecasting model a data-prior probability matrix (similar to that in Table 3.6) is formed by pooling together the data-prior vector for the months of the M.S. Matrices of this nature were calculated for corn and soybeans (see data-prior probability tables in the Appendix).

The Posterior Probabilities: All the elements defined up to Section 3.8 of Chapter III are already available. Using the Bayes Formula 3.6 (Section 3.8), the prediction posterior probability matrices of Table 3.3 were computed. A total of $50 \mathbb{P} \mathbb{P} \mathbb{P}_{k}$ matrices were obtained, five matrices per prediction model (see $\mathbb{P} \mathbb{P} \mathbb{P}_{k}$ tables in the Appendix).
D. The Model Results for the 1976-1977 Marketing Seasons of Corn and Soybeans

Three Bayesian decision models are reported; the NONDATA approach made use of the two types of prior probabilities; "data" and "nondata". The DATA approach combined sample or forecast information with the nondata-prior distributions in order to select the marketing action that maximizes expected gain (the PDWM reported in the introduction). All the price prediction models appearing in Chapter IV provided the sample information to the decision model. The results of the MAPM model are included even though it fails to pass the basic hypothesis of Section 3.5.

1. Bayesian "NONDATA" strategies

The marketing decision was simulated for 1976-77 marketing season; first, a separate Bayesian model was computed for each nondata prior, that is, the same nondata prior probability vector was assumed to prevail over the entire
marketing season. The process was repeated four times (one for each prior of Table 5.8). Second, the Bayesian model was calculated making use of the data-priors of the forecasting models. The process was repeated five times (one for each forecasting model).

The results of the nondata-priors appear in Tables 5.9 and 5.10. The upper half of the tables report the results from Equation 3.12. The main diagonal values of matrix $\mathbb{E} \mathbb{P}$ are shown in the columns of the tables (upper half) according to the four priors employed. The values represent expected prices for the months of the M.S. The lower half of the tables report the expected gains (or losses) per unit sold in the months of the M.S. These values are those of Equation 3.13. If one prior probability vector were chosen for the entire M.S., the relevant strategy is reported at the bottom of the tables. The action selected (selling month) was the one that maximized the expected gain of the column (lower half). A blank column could be left to the right of each table in which is entered the expected gain for each month that corresponds to the farmer's prior distribution for that month. The strategy of the farmer would be to identify among these selected values the month with the maximum expected gain per unit to be sold. The results of the data-priors are reported in a similar fashion in Tables 5.11 and 5.12.

Table 5.9. Bayesian "NONDATA" strategies using "NONDATA" priors, expected prices and expected returns per bushel of Iowa corn in 1976-77

| Months of the |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Expecte | ices ep $\left(a_{i}\right)$ |  |
| Dec. | 2.093 | 2.505 | 2.093 | 1.680 |
| Jan. | 2.141 | 2.658 | 2.141 | 1.623 |
| Feb. | 2.185 | 2.697 | 2.185 | 1.673 |
| Mar. | 2.224 | 2.697 | 2.224 | 1.751 |
| Apr. | 2.263 | 2.699 | 2.263 | 1.827 |
| May | 2.302 | 2.737 | 2.302 | 1.867 |
| June | 2.342 | 2.761 | 2.341 | 1.922 |
| July | 2.381 | 2.850 | 2.381 | 1.912 |
| Aug. | 2.420 | 2.947 | 2.420 | 1.894 |
| Sep. | 2.460 | 2.956 | 2.460 | 1.963 |
|  |  | Expected Gains eg ( $\mathrm{a}_{\mathrm{i}}$ ) |  |  |
| Dec. | 0.0 | 0.4124 | -9.5(10 ${ }^{-7}$ ) | -0.4124 |
| Jan. | $-6.1\left(10^{-5}\right)$ | 0.5172 | $7.1\left(10^{-5}\right)$ | -0.5173 |
| Feb. | $2.0\left(10^{-5}\right)$ | 0.5124 | $1.9\left(10^{-5}\right)$ | -0.5124 |
| Mar. | -9.5(10 ${ }^{-7}$ ) | 0.4733 | $-9.5\left(10^{-5}\right)$ | -0.4733 |
| Apr. | -9.5(10 $\left.{ }^{-7}\right)$ | 0.4357 | $-9.5\left(10^{-5}\right)$ | -0.4357 |
| May | -2.1(10 ${ }^{-5}$ ) | 0.4349 | $-2.1\left(10^{-5}\right)$ | -0.4349 |
| June | $-4.0\left(10^{-5}\right)$ | 0.4195 | $-3.0\left(10^{-5}\right)$ | -0.4196 |
| July | -4.1(10 ${ }^{-5}$ ) | 0.4690 | $-3.0\left(10^{-5}\right)$ | -0.4691 |
| Aug. | $-9.5\left(10^{-7}\right)$ | 0.5262 | $-9.5\left(10^{-7}\right)$ | -0.5262 |
| Sep. | $2.0\left(10^{-5}\right)$ | 0.4966 | $1.9\left(10^{-5}\right)$ | -0.4965 |
| Bayesian | $2.0\left(10^{-5}\right)$ | 0.5262 | I. 91 (10 $0^{-5}$ | Sell at |
| Solution | either |  | either | harvest |
| - | Feb. or Sep. | August | Feb. or S | time |

Table 5.10. Bayesian "NONDATA" strategies using "NONDATA" priors, expected prices and expected returns per bushel of Iowa soybeans in 1976-77

| Months of the marketing season |  | 0 | N | P |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Expected Prices (ep ( $\mathrm{a}_{\mathrm{i}}$ ) |  |  |
| Nov. | 5.916 | 6.867 | 5.916 | 4.965 |
| Dec. | 5.998 | 5.854 | 5.998 | 5.142 |
| Jan. | 6.974 | 6.905 | 6.074 | 5.243 |
| Feb. | 6.143 | 6.907 | 6.143 | 5.380 |
| Mar. | 6.213 | 6.948 | 6.213 | 5.478 |
| Apr. | 6.283 | 6.945 | 6.282 | 5.620 |
| May | 6.353 | 7.458 | 6.353 | 5.247 |
| June | 6.423 | 7.997 | 6.423 | 4.848 |
| July | 6.493 | 7.357 | 6.493 | 5.630 |
| Aug. | 6.564 | 7.917 | 6.564 | 5.211 |
|  |  | Expected Gains eg ( $\mathrm{a}_{\mathrm{i}}$ ) |  |  |
| Nov. | $1.72\left(10^{-5}\right)$ | 0.9508 | $6.67\left(10^{-6}\right)$ | -0.9508 |
| Dec. | $1.81\left(10^{-5}\right)$ | 0.8560 | $1.81\left(10^{-5}\right)$ | -0.8560 |
| Jan. | -2.19 (10 ${ }^{-5}$ ) | 0.8309 | -1.24 (10 ${ }^{-5}$ ) | -0.8309 |
| Feb. | $-6.29\left(10^{-5}\right)$ | 0.7631 | -4.29 (10 ${ }^{-5}$ ) | -0.7633 |
| Mar. | $-4.20\left(10^{-5}\right)$ | 0.7349 | $-6.20\left(10^{-5}\right)$ | -0.7350 |
| Apr. | $-1.03\left(10^{-4}\right)$ | 0.6624 | -1.02 (10 ${ }^{-4}$ ) | -0.6626 |
| May | $9.73\left(10^{-5}\right)$ | 1.1053 | $9.82\left(10^{-5}\right)$ | -1.1051 |
| June | $-2.19\left(10^{-5}\right)$ | 1.5744 | -1.24(10 ${ }^{-5}$ ) | -1.5744 |
| July | 1.72 (10 $0^{-5}$ ) | 0.8634 | $7.63\left(10^{-6}\right)$ | -0.8634 |
| Aug. | $1.72\left(10^{-5}\right)$ | 1.3530 | $6.67\left(10^{-6}\right)$ | -1.3530 |
| Bayesian Solution | 9.73 (10 $0^{-5}$ ) | 1.577 | $9.82\left(10^{-5}\right)$ | Sell at harvest time |
| Month | May | June | May |  |

Table 5.11. Bayesian "NONDATA" strategies using "DATA" priors, expected prices and expected returns per bushel of Iowa corn in 1976-77

| Months of marketing | $\begin{aligned} & \text { the } \\ & \text { season TPM } \end{aligned}$ | MAPM | TVLM | SEM | CBT-F |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Expected Prices ep ( $\mathrm{a}_{\mathrm{i}}$ ) |  |  |  |  |  |
| Dec. | 2.1276 | 1.9674 | 2.2269 | 2.1283 | 2.0928 |
| Jan. | 2.0676 | 2.0152 | 1.9527 | 1.9734 | 0.0 |
| Feb. | 1.8948 | 1.9156 | 1.9674 | 1.9466 | 0.0 |
| Mar. | 1.7549 | 1.8890 | 1.9179 | 1.8508 | 1.8899 |
| Apr. | 1.9895 | 2.0782 | 1.9639 | 1.9639 | 0.0 |
| May | 2.2317 | 2.2757 | 2.0036 | 2.0211 | 2.1791 |
| June | 2.3247 | 2.3161 | 2.0791 | 2.1128 | 0.0 |
| July | 2.0962 | 2.2196 | 2.0587 | 2.0685 | 1.9735 |
| Aug. | 2.0799 | 1.9946 | 1.9310 | 2.0057 | 0.0 |
| Sept. | 2.0988 | 1.8272 | 0.0 | 1.7273 | 0.0 |
| Expected Gains (eg ( $\mathrm{a}_{\mathrm{i}}$ ) |  |  |  |  |  |
| Dec. | 0.0348 | -0.1254 | 0.1341 | 0.0355 | -0.0000 |
| Jan. | -0.0732 | -0.1256 | -0.1881 | -0.1674 |  |
| Feb. | -0.2902 | -0.2694 | -0.2176 | -0.2384 |  |
| Mar. | -0.4691. | -0.3350 | -0.3061 | -0.3732 | -0.3341 |
| Apr. | -0.2736 | -0.1849 | -0.2992 | -0.2992 |  |
| May | -0.0706 | -0.0266 | -0.2987 | -0.2812 | -0.1232 |
| June | -0.0169 | -0.0255 | -0.2625 | -0.2288 |  |
| July | -0.2848 | -0.1614 | -0.3223 | -0.3125 | -0.4075 |
| Aug. | -0.3405 | -0.4258 | -0.4894 | -0.4147 |  |
| Sept. | -0.3612 | -0.6328 |  | -0.7327 |  |
| Bayesian Solution | 0.0348 | Sell at harvest time | 0.1341 | 0.0355 | Sell at harvest time |
| Month | Dec. |  | Dec. | Dec. |  |

Table 5.12. Bayesian "NONDATA" strategies using "DATA" priors, expected prices and expected returns per bushel of Iowa soybeans in 1976-77
Month
mark
Nov.
Dec.
6.3601 5.2998 5.8401 6.0709

Jan.
$6.3064 \quad 5.7028 \quad 5.9633 \quad 0.0$

Feb.
6.2430
5.6708
5.8899
5.7739

Mar.
Apr.
May
6.2064
5.8498
$5.9887 \quad 0.0$
6.3324
6.0051
6.1541
6.0948
6.2964
6.1889
6.4578
0.0

June
6.2416
6.0397
6.4205
4.9925

July
6.2330
5.8170
7.5374
0.0

Aug.

Nov.
$6.0576 \quad 5.6374 \quad 6.1262 \quad 5.4143$
5.7721
5.2776
0.0
0.0
$\underline{\text { Expected Gains eg }\left(a_{i}\right)}$

Dec.
$0.4439-\frac{\text { Expected Gains eg }\left(a_{i}\right)}{-0.6164-0.0761 \quad 0.1547}$

Jan.
$0.3087-0.2949 \quad 0.0344$

Feb.
$0.1685-0.4037-0.1846-0.3006$

Mar.
$0.0628-0.2936-0.1549$
Apr.
$0.1194 \quad-0.2079-0.0589 \quad-0.1182$
May
$0.0137-0.0938 \quad 0.1751$

June
$-0.1110 \quad-0.3129 \quad 0.0679-1.3601$

July
-0.1899 -0.6059 1.1145

Aug.
$-0.4358-0.8560-0.3672-1.0791$
-0.7921 -1.2856

| Bayesian <br> Solution <br> - | 0.4439 | Sell at <br> harvest <br> time | I.1145 | 0.1547 |
| :--- | :--- | :--- | :--- | :--- |
| Month | Nov. |  | June | Nov. |

2. Bayesian "DATA" strategies

The marketing decision for the 1976-77 marketing season was simulated using the price prediction models of Chapter IV. A separate Bayesian model was empioyed for each combination of one of the four nondata-prior probability vectors and one price prediction model. That is, one price prediction model and one nondata-prior probability vector were assumed to prevail over the entire marketing season; one pair at a time. The process was repeated for all possible combinations of nondata-priors, prediction models, and commodities (50 times).

The results of the DATA approach appear in Tables 5.13, 5.15, 5.17, 5.19, and 5.20 for corn and 5.14, 5.16, 5.18 and 5.21 for soybeans. The upper half of the tables report the results from Equation 3.14. The main diagonal values of the matrices $e p_{k} k=1, \ldots .5$ were clustered into a matrix (10 x 5), ten values were then selected from the matrix according with the 1976-77 predictions of the forecasting model in question. The selected ep $\left(a_{i}^{*} / Z_{k_{*}}\right)$ values are shown in the columns of the tables (upper half) as they correspond to the nondata-prior probability vectors employed. The lower half of the tables report the expected gains (or losses) per unit sold in the months of the 1976-77 M.S. The expected gains of the tables are also the selected values of the $E G$

Table 5.13. TPM model Bayesian "DATA" strategies, expected prices and expected returns per bushel of Iowa corn (1976-77 marketing season)

| Month of the |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| marketing season | I | O | N | P |


| Dec. | 1.9270 | 2.0928 | 2.0928 | 2.0928 |
| :--- | :--- | :--- | :--- | :--- |
| Jan. | 2.1132 | 2.2921 | 2.1202 | 1.9443 |
| Feb. | 2.9617 | 3.2770 | 2.5413 | 2.2366 |
| Mar. | 2.6555 | 3.1235 | 2.4201 | 1.9428 |
| Apr. | 1.7071 | 2.0094 | 2.0094 | 1.6236 |
| May | 2.8389 | 3.2415 | 2.8110 | 1.9262 |
| June | 2.2139 | 2.5649 | 2.9900 | 1.9301 |
| July | 2.7364 | 2.9130 | 2.5804 | 2.3505 |
| Aug. | 1.9180 | 2.1808 | 2.1808 | 1.8333 |
| Sept. | 0.9129 | 0.9129 | 0.9129 | 0.9129 |


| Dec. | Expected Gains eg ( $\mathrm{a}_{\mathrm{i}}^{*} / \mathrm{Z}_{\mathrm{k}^{*}}$ ) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | -0.1658 | 0.0 | 0.0 | 0.0 |
| Jan. | -0.0276 | 0.153 | -0.0206 | -0.1965 |
| Feb. | 0.7767 | 1.0920 | 0.3563 | 0.0516 |
| Mar. | 0.4315 | 0.8995 | 0.1961 | -0.2812 |
| Apr. | -0.5560 | -0.2537 | -0.2537 | -0.6395 |
| May | 0.5366 | 0.9392 | 0.5087 | -0.3761 |
| June | -0.1277 | 0.2233 | -0.0426 | -0.4155 |
| July | 0.3554 | 0.5320 | 0.1994 | -0.0305 |
| Aug. | -0.5024 | -0.2396 | -0.2396 | -0.5871 |
| Sep. | -1.5471 | -1.5471 | -1.5471 | -1.5471 |
| Bayesian Solution | 0.7767 | 1.0920 | 0.5087 | 0.0516 |
| Month | Feb. | Feb. | May | Feb. |

Table 5.14. TPM model Bayesian "DATA" strategies, expected prices and expected returns per bushel of Iowa soybeans (1976-77 marketing season)

| Month of the marketing season |  | 0 | N | P |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\underline{\text { Expected Prices ep }\left(\mathrm{a}_{\mathrm{i}}^{*} / \mathrm{z}_{\mathrm{k}^{*}}\right)}$ |  |  |
| Nov. | 6.3051 | 6.5149 | 6.0960 | 5.9611 |
| Dec. | 6.6433 | 7.0399 | 6.3968 | 5.8259 |
| Jan. | 7.5828 | 7.7244 | 6.8614 | 6.8614 |
| Feb. | 7.5577 | 7.5874 | 7.0532 | 7.0532 |
| Mar. | 6.7193 | 7.3128 | 6.6811 | 5.6549 |
| Apr. | 6.4778 | 6.6138 | 6.4177 | 6.3015 |
| May | 6.6871 | 8.5169 | 6.5727 | 4.5743 |
| June | 6.4229 | 6.4229 | 6.4229 | 6.4229 |
| July | 7.2570 | 7.2570 | 7.2570 | 7.2570 |
| Aug. | 8.3364 | 8.6031 | 7.8782 | 7.0769 |
|  |  | Expected Gains eg $\left(a_{i}^{*} / Z_{k^{*}}\right)$ |  |  |
| Nov. | 0.3889 | 0.5987 | 0.1798 | 0.0449 |
| Dec. | 0.6456 | 1.0422 | 0.3991 | -0.1718 |
| Jan. | 1.5083 | 1.6499 | 0.7869 | 0.7869 |
| Feb. | 1.4141 | 1.4438 | 0.9096 | 0.9096 |
| Mar. | 0.4367 | 1.0302 | 0.3985 | -0.6277 |
| Apr. | 0.1951 | 0.3311 | 0.1350 | 0.0188 |
| May | 0.3345 | 2.1646 | 0.2201 | -1.7783 |
| June | 0.0 | 0.0 | 0.0 | 0.0 |
| July | 0.7636 | 0.7636 | 0.7636 | 0.7636 |
| Aug. | 1.7722 | 2.0389 | 1.3140 | 0.5127 |
| Bayesian <br> Solution | 1.7722 | 2.1646 | 1.3140 | 0.9096 |
| Month | Aug. | May | Aug. | Feb. |

Table 5.15. MAPM model Bayesian "DATA" strategies, expected prices and expected returns per bushel of Iowa corn (1976-77 marketing season)

| Month of the marketing season |  | 0 | N | P |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Expected Prices ep $\left(\mathrm{a}_{\mathrm{i}}^{*} / \mathrm{Z}_{\mathrm{k}}\right.$ ) |  |  |
| Dec. | 2.0776 | 2.3975 | 2.1414 | 1.7342 |
| Jan. | 1.9374 | 2.0100 | 2.0100 | 1.8739 |
| Feb. | 1.9959 | 2.0655 | 2.0655 | 1.9330 |
| Mar. | 2.1956 | 2.4727 | 2.2904 | 1.9179 |
| Apr. | 1.7973 | 2.3499 | 2.0958 | 1.4725 |
| May | 0.9472 | 0.9472 | 0.9472 | 0.9472 |
| June | 2.4855 | 2.5367 | 2.4309 | 2.4309 |
| July | 0.0 | 0.0 | 0.0 | 0.0 |
| Aug. | 2.2228 | 3.3133 | 2.1773 | 1.4402 |
| Sept. | 0.9129 | 0.9129 | 0.9129 | 0.9129 |
|  |  | Expected Gains eg ( $\mathrm{a}_{\mathrm{i}}^{*} / \mathrm{Z}_{\mathrm{k} *}$ ) |  |  |
| Dec. | -0.0152 | 0.3047 | 0.0486 | -0.3586 |
| Jan. | -0.2034 | -0.1308 | -0.1308 | -0.2669 |
| Feb. | -0.1871 | -0.1195 | -0.1195 | -0.2520 |
| Mar. | -0.0284 | 0.2487 | 0.0664 | -0.3061 |
| Apr. | -0.4658 | 0.0868 | -0.1673 | -0.7906 |
| May | -1.3551 | -1.3551 | -1.3551 | -1.3551 |
| June | 0.1951 | 0.1951 | 0.0893 | 0.0893 |
| July | 0.0 | 0.0 | 0.0 | 0.0 |
| Aug. | 0.8929 | 0.8929 | 0.2431 | -0.9802 |
| Sept. | -1.5471 | -1.5471 | -1.5471 | -1.5471 |
| Bayesian Solution $\qquad$ | 0.8929 | 0.8929 | 0.2431 | 0.0893 |
| Month | Aug. | Aug. | Aug. | June |

Table 5.16. MAPM model Bayesian "DATA" strategies, expected prices and expected returns per bushel of Iowa soybeans (1976-77 marketing season)

| Month of the marketing season |  | 0 | N | P |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Expected Prices ep ( $\mathrm{a}_{\mathbf{i}}^{*} / \mathrm{Z}_{\mathrm{k} *}$ ) |  |  |
| Nov. | 5.0753 | 5.0753 | 5.0753 | 5.0753 |
| Dec. | 7.1429 | 7.9438 | 6.5577 | 5.7595 |
| Jan. | 5.5552 | 6.6218 | 5.8345 | 4.7897 |
| Feb. | 5.5323 | 6.6496 | 5.7927 | 4.8241 |
| Mar. | 5.6066 | 7.1676 | 5.6161 | 4.7417 |
| Apr. | 7.4263 | 7.7282 | 7.2996 | 6.3367 |
| May | 5.8383 | 6.0036 | 6.0036 | 5.7072 |
| June | 9.1077 | 9.5112 | 8.3676 | 7.1761 |
| July | 8.1326 | 8.0597 | 7.8236 | 7.8236 |
| Aug. | 6.8666 | 8.1503 | 6.6212 | 5.4899 |
|  |  | $\underline{\text { Expected Gains eg }\left(a_{i}^{*} / Z_{\mathrm{k}^{*}}\right)}$ |  |  |
| Nov. | -0.8409 | -0.8409 | -0.8409 | -0.8409 |
| Dec. | 1.1452 | 1.9461 | 0.5600 | -0.2382 |
| Jan. | -0.5143 | 0.5473 | -0.2400 | -1.2848 |
| Feb. | -0.6113 | 0.5060 | -0.3509 | -1. 3195 |
| Mar. | -0.6760 | 0.8850 | -0.6665 | -1.5409 |
| Apr. | 1.1436 | 1.4455 | 1.0169 | 0.0540 |
| May | -0.5143 | -0.3490 | -0.3490 | -0.6454 |
| June | 2.6849 | 3.0883 | 1.9447 | 0.7532 |
| July | 1.6392 | 1.5663 | 1.3302 | 1.3302 |
| Aug. | 0.3024 | 1.5861 | 0.0570 | -1.0743 |
| Bayesian Solution - | 2.6849 | 3.0883 | 1.9447 | 1.3302 |
| Month | June | June | June | July |

Table 5.17. TVLM model Bayesian "DATA" strategies, expected prices and expected returns per bushel of Iowa corn (1976-77 marketing season)

| Month of the <br> marketing season | I | O | N |  |
| :--- | :--- | :--- | :--- | :--- |

Dec.
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.

Dec.
Jan.
Feb.
Mar.
Apr.
May
June
July
Aug.
Bayesian
Solution
Month
2.6446
1.2867

1. 8830
2.1423
2.2817
2.4305
2.3584
2.5140
2.1665
0.5518
-0.8541
$-0.3020$
-0.0817
0.0186
0.1282
0.0168
0.1330
$-0.2539$
0.5518

Dec.
Dec.
Dec.
Dec.

Table 5.18. TVLM model Bayesian "DATA" strategies, expected prices and expected returns per bushel of Iowa soybeans (1976-77 marketing season)


Table 5.19. SEM model Bayesian "DATA" strategies, expected prices and expected returns per bushel of Iowa corn (1976-77 marketing season)

| Months of the marketing season |  | 0 | N | P |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\underline{\text { Expected Prices ep }\left(\mathrm{a}_{\mathrm{i}}^{*} / \mathrm{Z}_{\mathrm{k}^{*}}\right)}$ |  |  |
| Dec. | 2.5562 | 2.6033 | 2.3464 | 2.3464 |
| Jan. | 1.2866 | 2.0281 | 1.7281 | 0.9941 |
| Feb. | 1.8830 | 2.2448 | 2.1108 | 1.6562 |
| Mar. | 2.0147 | 2.0845 | 2.0845 | 1.9576 |
| Apr. | 2.2631 | 2.3799 | 2.2631 | 2.1463 |
| May | 2.4305 | 2.4818 | 2.3792 | 2.3792 |
| June | 2.3568 | 2.4867 | 2.3526 | 2.2219 |
| July | 2.5138 | 2.9124 | 2.4314 | 2.0437 |
| Aug. | 2.2459 | 2.3130 | 2.3130 | 2.1820 |
| Sept. | 2.4600 | 2.4600 | 2.4600 | 2.4600 |
|  |  | $\underline{\text { Expected Gains eg ( } \mathrm{a}_{\mathrm{i}} / \mathrm{Z}_{\mathrm{k} *} \text { ) }}$ |  |  |
| Dec. | 0.4634 | 0.5105 | 0.2536 | 0.2536 |
| Jan. | -0.8542 | -0.1127 | -0.4127 | -1.1467 |
| Feb. | -0.3020 | 0.0598 | -0.0742 | -0.5288 |
| Mar. | -0.2093 | -0.1395 | -0.1395 | -0.2664 |
| Apr. | -0.0000 | 0.1168 | -0.0000 | -0.1168 |
| May | 0.1282 | 0.1795 | 0.0769 | 0.0769 |
| June | 0.0152 | 0.1451 | 0.0110 | -0.1197 |
| July | 0.1328 | 0.5314 | 0.0504 | -0.3373 |
| Aug. | -0.1745 | -0.1074 | -0.1074 | -0.2384 |
| Sept. | 0.0 | 0.0 | 0.0 | 0.0 |
| Bayesian Solution | 0.4634 | 0.5314 | 0.2536 | 0.2536 |
| Month | Dec. | July | Dec. | Dec. |

Table 5.20. CBT-F model Bayesian "DATA" strategies, expected prices and expected returns per bushel of Iowa corn (1976-77 marketing season)

| Month of the marketing season |  | 0 | N | P |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Expected Prices ep ( $\mathrm{a}_{\mathrm{i}}^{*} / \mathrm{Z}_{\mathrm{k}^{*}}$ ) |  |  |
| Dec. | 3.3777 | 2.3831 | 2.2851 | 2.2851 |
| Mar. | 1.9868 | 2.3401 | 2.1474 | 1.7582 |
| May | 2.4756 | 2.7377 | 2.3837 | 2.1114 |
| July | 2.0998 | 2.6815 | 2.1197 | 1.7885 |
|  |  | Expected Gains eg ( $\mathrm{a}_{\mathrm{i}}^{*} / \mathrm{z}_{\mathrm{k} *}$ ) |  |  |
| Dec. | 0.2518 | 0.2903 | 0.1923 | 0.1923 |
| Mar. | -0.2372 | 0.1161 | -0.0766 | -0.4658 |
| May | 0.1733 | 0.4354 | 0.0814 | -0.1909 |
| July | -0.2812 | 0.3005 | -0.2613 | -0.5925 |
| Bayesian <br> Solution | 0.2518 | 0.4354 | 0.1923 | 0.1923 |
| Month | Dec. | May | Dec. | Dec. |

Table 5.21. CBT-F model Bayesian "DATA" strategies, expected prices and expected returns per bushel of Iowa soybeans


Table 5.21 (Continued)

| Month of the marketing season |  | 0 | N | P |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\underline{\text { Expected Gains eg }\left(\mathrm{a}_{\mathrm{i}}^{*} / \mathrm{z}_{\mathrm{k}^{*}}\right)}$ |  |  |
| Nov. | 0.2803 | 0.3924 | 0.1682 | 0.1682 |
| Jan. | 0.5381 | 0.9111 | 0.3066 | -0.1716 |
| Mar. | 1.2119 | 1.6056 | 0.5732 | 0.1921 |
| May | -0.5372 | 1.0260 | -0.3424 | -1.6324 |
| July | -0.0152 | 0.8037 | -0.0074 | -0.8243 |
| Bayesian Solution | 1.2119 | 1.6056 | 0.5732 | 0.1921 |
| Month | Mar. | Mar. | Mar. | Mar. |

matrices in 3.16b according to the $1976-77$ price predictions of the models.

If one nondata prior probability vector were chosen for the entire M.S., the relevant strategy would be the one reported at the bottom of the tables. The action selected (selling month) was the one that maximized the gain of the column (lower half). As in the NONDATA approach, a blank column could have been left to the right of the tables where the values from the priors are to be repeated according to the farmer's expectations over the months of the M.S. The strategy of the farmer would be, again, to identify among these selected values the month with maximum expected gain per unit to be sold.

## E. The Actual Gains for the 1976-77 Marketing Season

As compared with the actual gains observed for the 197677 M.S., the resulting strategies in the previous section seem to acquire more relevance. Table 5.22 shows the actual gains of corn and soybeans observed in 1976-77 M.S.

Table 5.22. Monthly actual gains per bushel of corn and soybeans over the 1976-77 marketing season ${ }^{\text {a }}$

| Month | Corn ${ }^{\text {b }}$ | Soybeans ${ }^{\text {b }}$ |
| :---: | :---: | :---: |
| November | - | 0.144 |
| December | 0.127 | 0.542 |
| January | 0.169 | 0.636 |
| February | 0.115 | 0.746 |
| March | 0.056 | 1.497 |
| April | 0.007 | 3.027 |
| May | -0.102 | 2.847 |
| June | -0.252 | 1.977 |
| July | -0.541 | 0.267 |
| August | $-0.830^{\text {c }}$ | $-1.274^{\text {c }}$ |
| September | _d | - |
|  | $C P\left(\mathrm{ph}_{\mathrm{i}}\right)$ <br> shel of <br> d-month. |  |

The actual gain for marketing corn reached its maximum in January at 0.169 dollars per bushel. It also seems from the table that early months such as December, January, and February were the best time to market corn in 1976-77, since gains decline steadily thereafter becoming even negative after April. All the Bayesian strategies of the DATA approach were clearly for the early disposal of the corn, with the exception of the MAPM-model same which failed all the tests of predictability. The month most recommended by the DATA approach was December with an average expected gain of 0.336 dollars per bushel of corn. On the other hand, the NONDATA approach using data-priors strongly advocates for the marketing of the corn at the beginning of the 1976-77 season. The solutions of the two Bayesian approaches seem to conform well to the actual developments of the corn marketing season. The NONDATA approach using nondata-priors deserve no comments here as results entirely depend on the good (or bad) judgement of the decision maker.

The actual gain (1976-77) for marketing soybeans reached its maximum in April at 3.072 dollars per bushel. Gains for soybeans increased steadily from a low of 0.144 dollars per bushel in November to a high of 3.027 in April, thereafter gains seem to decrease rapidly. In general, the Bayesian solutions for soybeans of the DATA approach do not favor
early disposal of the grain, the most recommended strategies split even between marketing at the middle and at the end of the 1976-77 season. The results of the NONDATA approach using data priors do not seem to conform to the actual gains of soybeans, only the TVLM model suggests the month of June while the rest indicate as best strategy either November or harvest time. The results indicate that prior information is more relevant in the case of soybeans than in the case of corn, thus, the DATA approach which considers the farmer's prior information is a more sensible aid to marketing decision making for soybean producers than methods purely based on past information.

## F. Revision of Strategies Over the Marketing Season

Up to this point we have avoided the treatment of farmer's revision of his marketing decision. Its earlier inclusion would have only obscured the development of our presentation without foreseeable gain in generality, yet the matter deserves some attention to which we turn now.

Assume a farmer whose best marketing strategy lat harvest time) was to delay his marketing transaction until the very end of the marketing season. He would not be satisfied by storing and forgetting his grain until the season ends. On the contrary, he is likely to be continuously
revising his initial strategy. New and/or better information about the marketing season is likely to influence his initial expectations. The farmer may (later in the marketing season) validate a price forecasting model he disregarded at harvest time or else he may wish to assign new weights to the prices of the payoff matrix (change his prior probability distribution). Thus, a restatement of the initial decision problem is needed.

In Chapter III we state that the farmer will engage in post-harvest marketing activities only if the expected return from this activity surpasses the harvest return plus the intertemporal transfer cost of the commodity. Now, let us restate the problem as follows; the farmer at any point in time of the marketing season will be willing to further delay his marketing transaction if and only if a future expected return surpasses the current return plus the additional storage expenses (storage used in a broad sense).

Calling "e" any post-harvest point in time when a reevaluation of the marketing strategy is wanted, Equation 3.1 is replaced in the analysis by Equation 5.2.

$$
\begin{equation*}
\operatorname{Rbep}_{i}^{e}=C P\left(p h_{e}\right)(l+r)^{i-e}+s C_{i-e} \quad e, i=1,2 \ldots m \tag{5.2}
\end{equation*}
$$

where:
Rbep $e_{i}^{-}$revised break even price at the eth postharvest period for the ith post-harvest period

CP (phe) - the current cash price at the eth post-harvest period
r - rate of interest per ph period
$S C_{i-e}$ - cost of storage from $\mathrm{ph}_{\mathrm{e}}$ to $\mathrm{ph}_{\mathrm{i}}$
It is clear that with the new equation at hand we are able to carry out the entire analysis developed in Chapter III. Minor changes, though, are anticipated: 1) The courses of action open to the decision maker reduce in number as e approaches $m, 2$ ) the farmer's priors introduced in the model for the remaining post-harvest periods (after $\mathrm{ph}_{e}$ ) must reflect any change of the farmer initial expectations, if any, 3 ) the number of rows of the payoff matrix reduces with the number of courses of action available.

In some cases, the price prediction models can be actualized to the post-harvest reevaluation period as well. Models which based their predictions on the latest data available at harvest time may also provide new predictions on the latest data available at any point in time over the marketing season. In the trend price model, for example, if the difference between $C P(h)$ and $C P\left(p h_{i}\right)$ in marketing season $t-1$ is added to the harvest price $C P(h)$ in marketing season $t$; the resulting value is the price prediction of the model for the ith post-harvest period in marketing season $t$. Substituting $C P\left(\mathrm{ph}_{e}\right)$ for $C P(h)$ in the TPM model, we can have a set of new
price forecasts to be used in making a revision of marketing strategies at period phe Obviously, we have to generate as many price prediction sets such as that of Table 4.1 as the number of ph e periods considered in the revision process. As a matter of fact, for practical purposes we are dealing with an entirely new price prediction model.

The revision of strategies over the marketing season, as it can be seen, is purely a mechanical process of restatements (substitute $\mathrm{ph}_{\mathrm{e}}$ for h ) and adjustments (changes in the number of available actions). However, the process demands a tremendous amount of work if many phe periods are to be considered. If the model of this dissertation has any practical use at all, it would be worthwhile to build a versatile and comprehensive computer program of its basic structural form so that initial and revised strategies can readily be obtained by simple changes in the parameters.
VI. SUMMARY AND CONCLUSIONS
A. Summary

Providing professional advice in grain marketing is to a great extent a hazardous job. The nature of the marketing forces are such that good prospects of large gains by selling on one specific month of the M.S. will reverse if all farmers act according to this expectation. It is conceivable to say that a grain marketing advice has to be partially believed and followed among farming units in order to become true.

The primary objective of this dissertation was to develop an economic model that would allow us to assist the grain producer on his marketing decision without telling him what is best to do. The farmer decides on the price forecasting model which seems to make more sense to him. Also, he specifies his "current feelings" about a set of prices which are likely to come. The price forecast made by the model of his choice is then double-adjusted; first, by the past performance of the forecasting model itself, and second, by the "current feelings of the farmer". The resulting expected prices and expected gains are then provided to the farmer.

The economic model developed in this dissertation incorporates Bayesian Decision Theory with price forecasting models. The objective was to picture, as closely as possible, the marketing problem of the grain producer with the un-
certainty elements, alternative options, and course of action experienced in the real world.

Basic to the Bayesian decision model was the development of theoretical analysis of price patterns. The rational expectation hypothesis was used to establish that break-even prices are the values of central tendency of the cash prices over a number of years. A feasible range of cash prices was determined for each one of the months of the marketing season. The distributions of the monthly cash prices about their bep counterpart were employed to locate the values on the boundary of the range. Definition of the feasible range eliminates from consideration irrelevant alternatives.

The states of nature considered in the Bayesian model
were the intervals of the feasible range that are equally likely, mutually exclusive, and collectively exhaustive. The payoff matrix was defined with mid-range values of the states of nature. Two types of Bayesian prior probability distributions were defined; the "data" priors were based on the relative frequencies over the states of nature of sample data of five forecasting models. The "nondata" priors were based on four assumptions of the farmer's attitudes (current feelings). towards the alternative prices of the payoff matrix. Conditional probability distributions over the states of nature were also obtained from the price forecasting models, these probabilities were obtained by calculating the empirical relative
frequency with which the respective pairs of outcomes and predictions have occurred over the states of nature since 1955. The conditional and the nondata prior probabilities were combined by use of Bayes' Theorem to compute a posterior probability of observing each state of nature given a specific price prediction. The process was repeated for the following price forecasting models; Trend-Price Model (TPM), Moving-Average Price Model (MAPM), Two-Variable Linear Model (TVLM), Single-Equation Model (SEM), and Futures Market Model (CBT-F).

The "data" prior probabilities of the forecasting models and each one of the assumed "nondata" prior probabilities were utilized to select the grain marketing action (selling month) that maximized expected gains in the Bayesian NONDATA approach. The Bayesian DATA approach utilized the prediction posterior probabilities to calculate the expected gains. In this last case, Bayesian solutions were provided for all possible combinations of "nondata" priors and price forecasting models. On either approach, DATA and NONDATA, the strategy selected was the strategy that maximized the price differential within the market (PDWM); that is, the expected gain based on the Iowa State average data.

Associated with the results of the Bayesian decision model were the testing of central tendency and equal variance of the sample distributions of the price fore-
casting models. We established that an efficient price forecasting model must not make by definition irrelevant predictions (values outside the feasible range) nor must it exclude from consideration relevant alternatives (subsets of the feasible range). This analysis was extremely relevant by itself as it provides grounds to compare the expected performance of the forecasting models. All the prediction models studied in this dissertation passed the testing of the above hypotheses with the exception of the moving-average price model which fails to be comprehensive of the feasible range of the cash prices and has a biasness factor significantly different from zero.

## B. Results and Conclusions

The Bayesian expected price differentials within the market (PDWM) were estimated in Chapter $V$, a summary of the Bayesian solutions is reproduced here in Tables 6.1 and 6.2.

The Bayesian solutions for corn of the DATA approachnondata priors seem to suggest early disposal of the grain for the 1976-77 marketing season. Four of the five forecasting models are for early months such as December or February to sell the corn regardless of the farmer's current feelings. Only, the MAPM model (this model failed the tests of predictability) advises late disposal of the grain, August if

Table 6.1. Summary of the Bayesian solutions for the 1976-77 marketing season of Iowa corn (expected gains in dollars per bushel)

| Bayesian Decision Model | Nondata-priors |  |  |  | Data-priors of the forecasting |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Data-approach | $\bar{I}$ | 0 | N | P | models |
| TPM | 0.7767 | 1.0920 | 0.5087 | 0.0516 | 0.0348 |
|  | February | February | May | February | December |
| MAPM | 0.8929 | 0.8929 | 0.2421 | 0.0833 | Sell at |
|  | August | August | August | June | harvest <br> time |
| TVLM | 0.5518 | 0.5971 | 0.9023 | 0.9023 | 0.1341 |
|  | December | December | December | December | December |
| SEM | 0.4634 | 0.5314 | 0.2536 | 0.2536 | 0.0355 |
|  | December | July | December | December | December |
| CBT-F | 0.2518 | 0.4354 | 0.1923 | 0.1923 | Sell at harvest time |
|  | December | May | December | December |  |
| Nondata- | $2.0\left(10^{-5}\right)$ | 0.5262 | $1.9\left(10^{-5}\right)$ | Sell at |  |
| approach | Feb. or | August | Feb, or | harvest |  |


farmer's current feelings are somehow at least not pessimistic and June if they are. The month most recommended by the prediction models is December with an average expected gain of 0.3360 cents per bushel of corn.

The solutions of the NONDATA approach with the data priors of the forecasting models also suggest the initial months of the marketing season as the best strategies to sell corn in 1976-77. To sell at harvest time or during the month of December are the courses of action suggested by the solutions with expected marketing gains no greater than 0.0355 cents per bushel of corn. The solutions of the NONDATA approach with nondata priors vary according to the farmer's marketing attitudes revealed by the nondata priors. Optimistic views such as prior "O" plead in favor of selling the grain close to the end of the 1976-77 marketing season. In contrast, prior "P" is for selling the corn at harvest time. Intermediate priors such as "I" and "N" show mixed strategies with very small expected gains.

In general, the Bayesian solutions for soybeans of the DATA approach-nondata priors do not favor early disposal of the grain for the 1976-77 marketing season. Early months such as November appear only once in the solutions (TVCMmodel, prior "I"), while months such as June, July, and August appear seven times. Months located at the middle of the
marketing season are also favored by the 1976-77 Bayesian solutions of the DATA approach.

The solutions of the NONDATA approach using the data priors of the forecasting models do not seem to support the DATA approach solutions. Only the TVLM model suggest to dispose of the grain late in the marketing season (June). The other three solutions advise either harvest time or as early as November to dispose of the farmer's soybeans.

The solutions of the NONDATA approach using the nondata priors suggested in Chapter $V$ follow in the case of soybeans the same pattern of the corn solutions. A person having optimistic "feeling" about the 1976-77 marketing season can expect marketing gains up to $\$ 1.5744$ per bushel of soybeans by delaying the marketing transaction until June, while a pessimist may not expect either marketing gain or loss by selling at harvest time. Marketing "feelings" between the two extremes may expect almost negligible marketing gains or losses with May as the best month to sell soybeans.

Five price forecasting models have been studied, four assumptions about "farmer's current feelings on prices" have been made. For an Iowa producer of corn and soybeans who bases his marketing decisions on at least one of the above models or assumptions, the model of this dissertation has provided expected prices and expected gains for the 1976-77 marketing
season, also it has provided a strategy or course of action which maximizes the expected market return to the farmer. Yet we recognize the need for further research in at least two directions; l) we have only considered the marketing decision problem at harvest time, it would be of interest to expand the model in a way that continuous revision of marketing decisions is possible. Such expansion can be made possible by assuming that the whole analysis of Chapter III beginning with the bep Equation 3.1 uses $C P(h)$ at harvest time, $C P\left(p h_{1}\right)$ a month later, $C P\left(\mathrm{ph}_{2}\right)$ two months later, and so on. Clearly, the courses of action available to the decision maker reduce by one each month. 2) We have analyzed five price forecasting models and four nondata priors, it would also be of interest to analyze many other alternative forecasting models and priors. Such enlargement means to us; on one hand, more specific answers to specific farmer's marketing problems, and on the other, a more comprehensive view of forecasts to the analyst.

In many cases there is the problem of systematically comparing alternative hypotheses; for example, the investigator may be interested in providing to the decision maker some guidance for comparison between two or more forecasting models, or simply he may wish to compare the efficiency of different nondata prior distributions to specific forecasting
models. The model of this dissertation can also be an aid in comparing and testing these kinds of hypotheses. We recognize though that in order to better serve this purpose more research is needed on the feasible range of the outcomes. Also an important step towards meaningful comparisons would be to define a coefficient of predictability based on the expected distribution of the Prediction Posterior Probability matrices of a hypothetical perfect forecasting model.
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IX. APPENDIX

Table A.l. Average prices of com received by farmers in the state of Iowa (l5th day of each month)

| Marketing month | Dec. | Jan. | Feb. | Mar. | Apr | May | June | July |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1950-51 |  | 1.51 | 1.55 | 1.56 | 1.58 | 1.58 | 1.56 | 1.58 |
| 1951-52 | 1.41 | 1.60 | 1.54 | 1.54 | 1.60 | 1.63 | 1.68 | 1.66 |
| 1952-53 | 1.41 | 1.39 | 1.33 | 1.37 | 1.38 | 1.41 | 1.38 | 1.39 |
| 1953-54 | 1.39 | 1.39 | 1.40 | 1.40 | 1.41 | 1.44 | 1.46 | 1.45 |
| 1954-55 | 1.36 | 1.36 | 1.35 | 1.30 | 1.33 | 1.38 | 1.40 | 1.40 |
| 1955-56 | 1.22 | 1.21 | 1.21 | 1.21 | 1.33 | 1.41 | 1.43 | 1.44 |
| 1956-57 | 1.20 | 1.19 | 1.14 | 1.14 | 1.18 | 1.20 | 1.21 | 1.20 |
| 1957-58 | 0.85 | 0.78 | 0.78 | 0.83 | 1.00 | 1.07 | 1.12 | 1.09 |
| 1958-59 | 0.97 | 0.97 | 0.97 | 1.00 | 1.06 | 1.08 | 1.09 | 1.06 |
| 1959-60 | 0.83 | 0.84 | 0.82 | 0.84 | 0.92 | 0.96 | 1.03 | 1.04 |
| 1960-61 | 0.84 | 0.90 | 0.93 | 0.93 | 0.90 | 0.98 | 1.00 | 1.01 |
| 1961-62 | 0.87 | 0.88 | 0.89 | 0.91 | 0.93 | 0.95 | 0.98 | 1.00 |
| 1962-63 | 0.92 | 0.98 | 0.99 | 0.99 | 1.00 | 1.04 | 1.11 | 2.14 |
| 1963-64 | 1.01 | 1.02 | 1.02 | 1.06 | 1.11 | 1.12 | 1.10 | 1.06 |
| 1964-65 | 1.09 | 1.09 | 1.10 | 1.10 | 1.14 | 1.16 | 1.18 | 1.15 |
| 1965-66 | 1.02 | 1.07 | 1.07 | 1.03 | 1.09 | 1.13 | 1.14 | 1.21 |
| 1966-67 | 1.25 | 1.24 | 1.20 | 1.02 | 1.20 | 1.21 | 1.23 | 1.18 |
| 1967-68 | 0.99 | 1.00 | 1.01 | 1.02 | 1.05 | 1.08 | 1.06 | 1.02 |
| 1968-69 | 1.02 | 1.06 | 1.06 | 1.05 | 1.07 | 1.15 | 1.14 | 1.14 |
| 1969-70 | 1.04 | 1.07 | 1.06 | 1.05 | 1.08 | 1.12 | 1.15 | 1.18 |
| 1970-71 | 1.31 | 1.36 | 1.38 | 1.36 | 1.34 | 1.32 | 1.38 | 1.31 |
| 1971-72 | 1.05 | 1.04 | 1.04 | 1.05 | 1.08 | 1.10 | 1.09 | 1.10 |
| 1972-73 | 1.35 | 1.30 | 1.26 | 1.28 | 1.31 | 1.51 | 1.93 | 1.96 |
| 1973-74 | 2.31 | 2.51 | 2.67 | 2.59 | 2.30 | 2.40 | 2.53 | 2.91 |
| 1974-75 | 3.24 | 3.01 | 2.82 | 2.63 | 2.65 | 2.67 | 2.65 | 2.70 |
| 1975-76 | 2.30 | 2.37 | 2.43 | 2.44 | 2.42 | 2.59 | 2.69 | 2.78 |
| 1976-77 | 2.22 | 2.31 | 2.30 | 2.28 | 2.27 | 2.20 | 2.09 | 1.84 |
| Source: U.S.D.A. Economic Research Service. Agricultural prices, Crop Reporting Board, Washington D.C., annual summaries 19501977. |  |  |  |  |  |  |  |  |


| Aug. | Sept. | Oct. | Nov. |
| :--- | :--- | :--- | :--- |
| 1.60 | 1.61 | 1.60 | 1.61 |
| 1.63 | 1.61 | 1.43 | 1.35 |
| 1.40 | 1.44 | 1.29 | 1.30 |
| 1.47 | 1.49 | 1.44 | 1.32 |
| 1.32 | 1.29 | 1.16 | 1.19 |
|  |  |  |  |
| 1.47 | 1.44 | 1.21 | 1.21 |
| 1.18 | 1.07 | 1.00 | 0.92 |
| 1.09 | 1.05 | 0.98 | 0.86 |
| 1.07 | 1.02 | 0.97 | 0.90 |
| 1.01 | 1.00 | 0.96 | 0.75 |
|  |  |  |  |
| 1.00 | 0.99 | 0.96 | 0.87 |
| 0.96 | 0.98 | 0.98 | 0.85 |
| 1.13 | 1.16 | 1.01 | 0.97 |
| 1.07 | 1.10 | 1.05 | 1.00 |
| 1.11 | 1.11 | 1.05 | 0.95 |
|  |  |  |  |
| 1.27 | 1.28 | 1.23 | 1.20 |
| 1.06 | 1.06 | 1.02 | 0.95 |
| 0.96 | 0.97 | 0.97 | 1.00 |
| 1.13 | 1.09 | 1.05 | 1.04 |
| 1.21 | 1.31 | 1.24 | 1.24 |
| 1.13 | 1.01 | 0.94 | 0.94 |
| 1.09 | 1.16 | 1.10 | 1.14 |
| 2.65 | 2.00 | 2.06 | 2.14 |
| 3.34 | 3.26 | 3.44 | 3.27 |
| 2.94 | 2.76 | 2.54 | 2.30 |
| 2.60 | 2.60 | 2.27 | 2.01 |
| 1.59 |  |  |  |
|  |  |  |  |
| 1.09 |  |  |  |

Table A, 2. Average prices of soybeans received by farmers in the state of Iowa (15th day of each month)

| Marketing <br> month | Dec. | Jan. | Feb. | Mar. | Apr. | May | June | July | Aug. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1950-51$ |  | 2.87 | 3.09 | 3.13 | 3.13 | 3.14 | 2.94 | 2.83 | 2.76 |
| $1951-52$ | 2.73 | 2.76 | 2.75 | 2.72 | 2.64 | 2.60 | 2.97 | 2.95 | 3.06 |
| $1952-53$ | 2.73 | 2.67 | 2.59 | 2.77 | 2.78 | 2.74 | 2.63 | 2.41 | 2.38 |
| $1953-54$ | 2.81 | 2.80 | 2.95 | 3.18 | 3.51 | 3.57 | 3.48 | 3.40 | 3.20 |
| $1954-55$ | 2.53 | 2.52 | 2.55 | 2.47 | 2.35 | 2.28 | 2.27 | 2.15 | 2.14 |
| $1955-56$ | 2.09 | 2.19 | 2.23 | 2.35 | 2.61 | 2.96 | 2.87 | 2.41 | 2.37 |
| $1956-57$ | 2.26 | 2.30 | 2.21 | 2.24 | 2.23 | 2.18 | 2.14 | 2.18 | 2.21 |
| $1957-58$ | 2.00 | 2.01 | 2.00 | 2.06 | 2.12 | 2.10 | 2.08 | 2.06 | 2.03 |
| $1958-59$ | 2.02 | 2.03 | 2.04 | 2.06 | 2.08 | 2.10 | 2.07 | 2.04 | 1.96 |
| $1959-60$ | 1.92 | 1.94 | 1.93 | 1.93 | 1.96 | 1.92 | 1.90 | 1.89 | 1.97 |
|  |  |  |  |  |  |  |  |  |  |
| $1950-61$ | 1.95 | 2.22 | 2.48 | 2.61 | 3.04 | 2.94 | 2.63 | 2.55 | 2.53 |
| $1961-62$ | 2.33 | 2.34 | 2.32 | 2.34 | 2.36 | 2.33 | 2.31 | 2.32 | 2.31 |
| $1962-63$ | 2.32 | 2.36 | 2.42 | 2.43 | 2.38 | 2.42 | 2.43 | 2.39 | 2.37 |
| $1963-64$ | 2.53 | 2.60 | 2.54 | 2.51 | 2.42 | 2.31 | 2.32 | 2.31 | 2.31 |
| $1954-65$ | 2.68 | 2.70 | 2.75 | 2.81 | 2.79 | 2.65 | 2.68 | 2.65 | 2.49 |
|  |  |  |  |  |  |  |  |  |  |
| $1955-66$ | 2.47 | 2.62 | 2.71 | 2.66 | 2.74 | 2.86 | 3.02 | 3.38 | 3.53 |
| $1956-67$ | 2.85 | 2.77 | 2.68 | 2.71 | 2.67 | 2.65 | 2.67 | 2.63 | 2.52 |
| $1967-68$ | 2.49 | 2.52 | 2.55 | 2.54 | 2.54 | 2.56 | 2.52 | 2.51 | 2.51 |
| $1968-69$ | 2.41 | 2.44 | 2.45 | 2.45 | 2.49 | 2.54 | 2.49 | 2.50 | 2.49 |
| $1969-70$ | 2.23 | 2.31 | 2.35 | 2.37 | 2.44 | 2.49 | 2.56 | 2.70 | 2.62 |
|  |  |  |  |  |  |  |  |  |  |
| $1970-71$ | 2.60 | 2.80 | 2.86 | 2.85 | 2.73 | 2.79 | 2.95 | 3.17 | 3.07 |
| $1971-72$ | 2.94 | 2.90 | 2.97 | 3.14 | 3.35 | 3.34 | 3.30 | 3.32 | 3.37 |
| $1972-73$ | 3.99 | 4.12 | 5.44 | 6.02 | 6.11 | 8.25 | 10.10 | 6.65 | 9.30 |
| $1973-74$ | 5.60 | 5.80 | 6.00 | 5.85 | 5.05 | 5.15 | 5.09 | 6.09 | 7.52 |
| $1974-75$ | 7.13 | 6.25 | 5.73 | 5.30 | 5.63 | 4.95 | 4.91 | 5.25 | 5.82 |
| $1975-76$ |  |  |  |  |  |  |  |  |  |
| $1976-77$ | 6.54 | 4.41 | 4.43 | 4.37 | 4.45 | 4.88 | 6.17 | 6.69 | 6.08 |
| 1 | 6.89 | 7.71 | 9.31 | 9.20 | 8.40 | 6.76 | 5.29 |  |  |

Source: U.S.D.A. Economic Research Service. Agricultural prices, Crop Reporting Board, Washington D.C., annual summaries 1950-1977. $a_{\text {Preliminary mid-month }}$.

| Sept. | Oct. | Nov. |
| :--- | :--- | :--- |
| 2.62 | 2.61 | 2.76 |
| 2.85 | 2.69 | 2.71 |
| 2.33 | 2.40 | 2.59 |
| 2.45 | 2.48 | 2.53 |
| 1.98 | 2.05 | 2.07 |
|  |  |  |
| 2.05 | 2.08 | 2.26 |
| 2.07 | 2.00 | 1.99 |
| 1.94 | 1.92 | 1.94 |
| 1.89 | 1.91 | 1.97 |
| 1.90 | 1.88 | 1.89 |
|  |  |  |
| 2.28 | 2.17 | 2.29 |
| 2.29 | 2.20 | 2.27 |
| 2.40 | 2.50 | 2.63 |
| 2.45 | 2.49 | 2.54 |
| 2.43 | 2.28 | 2.34 |
|  |  |  |
| 2.90 | 2.77 | 2.82 |
| 2.51 | 2.44 | 2.44 |
| 2.45 | 2.33 | 2.40 |
| 2.32 | 2.19 | 2.22 |
| 2.61 | 2.71 | 2.80 |
|  |  |  |
| 2.90 | 2.94 | 2.84 |
| 3.31 | 3.06 | 3.40 |
| 5.75 | 5.49 | 5.10 |
| 7.34 | 8.19 | 7.45 |
| 5.35 | 4.88 | 4.48 |
|  |  |  |
| 6.62 | 5.80 | 6.06 |
|  |  |  |

Table A.3. Interest rates paid by farmers:

| Marketing season | Monthly interest rate (r) | Annual interest rate |
| :---: | :---: | :---: |
| 1955-56 | 0.38347 | 4.7 |
| 1956-57 | 0.38347 | 4.7 |
| 1957-58 | 0.39146 | 4.8 |
| 1958-59 | 0.39944 | 4.9 |
| 1959-60 | 0.40741 | 5.0 |
| 1960-61 | 0.41667 | 5.1 |
| 1961-62 | 0.42334 | 5.2 |
| 1962-63 | 0.43129 | 5.3 |
| 1963-64 | 0.43129 | 5.3 |
| 1964-65 | 0.43927 | 5.4 |
| 1965-66 | 0.43927 | 5.4 |
| 1966-67 | 0.43927 | 5.4 |
| 1967-68 | 0.45510 | 5.6 |
| 1968-69 | 0.46302 | 5.7 |
| 1969-70 | 0.47094 | 5.8 |
| 1970-71 | 0.48675 | 6.0 |
| 1971-72 | 0.50254 | 6.2 |
| 1972-73 | 0.51830 | 6.4 |
| 1973-74 | 0.53403 | 6.6 |
| 1974-75 | 0.58888 | 7.3 |
| 1975-76 | 0.68215 | 8.5 |
| 1976-77 | $0.68215^{\text {a }}$ | $8.5{ }^{\text {a }}$ |
| Source: U.S.D.A. Economic Research Service. Agricultural Finance Tables (farm mortgage interest rates, Table 24, Vol. 1960-1976) |  |  |

Table A.4. Handling and storage cost for corn and soybeans in commercial Iowa elevators

| Years | Carried in S |  | torage and Conditioning Cost for Number of Months |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Carloads | Others | 1 | 2 | 3 | 4 |
| Corn and Soybeans |  |  |  |  |  |  |
| 1955 | 0.310345 | 4.488505 | 1.454826 | 1.455630 | 1.447584 | 1.422987 |
| 1956 | 0.108696 | 4.413043 | 1.615216 | 1.496521 | 1.496521 | 1.452173 |
| 1957 | 0.545454 | 4.636363 | 1.480604 | 1.482726 | 1.482726 | 1.441514 |
| 1958 | 0.196967 | 4.590910 | 1.497271 | 1.497271 | 1.492120 | 1.462423 |
| 1959 | 0.256098 | 4.707316 | 1.495364 | 1.495364 | 1.470974 | 1.434144 |
| 1960 | 0.277777 | 4.314815 | 1.489627 | 1.489627 | 1.471108 | 1.439998 |
| 1961 | 0.343750 | 4.109375 | 1.467187 | 1.467187 | 1.467187 | 1.435937 |
| 1962 | 0.519231 | 2.500000 | 1.483076 | 1.483076 | 1.483076 | 1.444230 |
| 1963 | 0.700581 | 2.501743 | 1.467323 | 1.467323 | 1.441742 | 1.388719 |
| 1964 | 1.046296 | 2.362962 | 1.198888 | 1.198888 | 1.169258 | 1.162962 |
| 1965 | 0.380000 | 2.200000 | 1.246199 | 1.246199 | 1.206199 | 1.191399 |
| 1966 | 0.597826 | 2.576086 | 1.248694 | 1.181303 | 1.170433 | 1.152172 |
| 1967 | 0.714286 | 2.158928 | 1. 307320 | 1.298391 | 1.235891 | 1.121964 |
| 1968 | 0.485714 | 2.214285 | 1.402569 | 1.402569 | 1.402569 | 1.227998 |
| 1969 | 1.000000 | 1.846153 | 1.335383 | 1.329999 | 1.412306 | 1.138461 |
| 1970 | 0.000000 | 2.250000 | 1. 388332 | 1.388332 | 1.263333 | 1.208333 |
| Corn Only |  |  |  |  |  |  |
| 1971 | 0.625000 | 2.312500 | 1.696250 | 1.371249 | 1.371249 | 1.237499 |
| 1972 | 0.333330 | 2.166660 | 1.468333 | 1.468333 | 1.384999 | 1.150000 |
| 1973 | 1.257575 | 3.742424 | 2.021813 | 2.021813 | 2.021813 | 1.592422 |
| 1974 | 2.272727 | 4.181818 | 2.141816 | 2.141816 | 2.096361 | 1.522727 |
| 1975 | 2.133330 | 3.866660 | 2.163996 | 2.163996 | 2.097330 | 1.516666 |
| 1976 | 2.500000 | 3.916666 | 2.429162 | 2.429162 | 2.429162 | 1.926666 |
| 1977 | 2.000000 | 3.500000 | 3.412499 | 3.412499 | 3.037499 | 2.500000 |
| Soybeans Only |  |  |  |  |  |  |
| 1971 | 0.833333 | 2.416666 | 1.766666 | 1.333333 | 1.333333 | 1.333333 |
| 1972 | 0.333330 | 2.166666 | 1.523333 | 1.523333 | 1.440000 | 1.250000 |
| 1973 | 1.378787 | 4.515151 | 2.408175 | 2.408175 | 2.408175 | 1.890908 |
| 1974 | 2.818181 | 5.000000 | 2.814543 | 2.814543 | 2.814543 | 1.918180 |
| 1975 | 2.133330 | 4.400000 | 2.440662 | 2.440662 | 2.373996 | 1.700000 |
| 1976 | 2.916666 | 4.500000 | 2.680829 | 2.680829 | 2.680829 | 2.051666 |
| 1977 | 2.000000 | 3.500000 | 4.164997 | 4.164997 | 3.665000 | 2.875000 |


| 5 |  | Storage and Conditioning Cost for Number of Months |  |  |  | $\frac{\text { Months }}{11}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.422987 | 1.406896 | 1.406896 | 1.406896 | 1.406896 | 1.406896 | 1.406896 |
| 1.452173 | 1.443477 | 1.443477 | 1.443477 | 1.443477 | 1.443477 | 1.443477 |
| 1.436362 | 1.424241 | 1.424241 | 1.424241 | 1.424241 | 1.424241 | 1.424241 |
| 1.457575 | 1.451514 | 1.451514 | 1.451514 | 1.451514 | 1.451514 | 1.451514 |
| 1.421949 | 1.375608 | 1.376608 | 1.375608 | 1.375608 | 1.375608 | 1.375608 |
| 1.421479 | 1.425183 | 1.425183 | 1.425183 | 1.425183 | 1.425183 | 1.425183 |
| 1.428124 | 1.415625 | 1.415625 | 1.415625 | 1.415625 | 1.415625 | 1.415625 |
| 1.423076 | 1.400000 | 1.400000 | 1.400000 | 1.400000 | 1.400000 | 1.400000 |
| 1.389300 | 1.355580 | 1.355580 | 1.355580 | 1.353254 | 1.353254 | 1.353254 |
| 1.147406 | 1.147406 | 1.147406 | 1.147406 | 1.147406 | 1.147406 | 1.147406 |
| 1.147199 | 1.143198 | 1.143198 | 1.143198 | 1.143198 | 1.143198 | 1.143198 |
| 1.152172 | 1.143476 | 1.143476 | 1.154346 | 1.154346 | 1.154346 | 1.154346 |
| 1.115355 | 1.108213 | 1.108213 | 1.108213 | 1.105177 | 1.105177 | 1.105177 |
| 1.209712 | 1.198283 | 1.198283 | 1.198283 | 1.198283 | 1.198283 | 1.198283 |
| 1.100000 | 1.100000 | 1.100000 | 1.100000 | 1.100000 | 1.100000 | 1.100000 |
| 1.224999 | 1.225000 | 1.225000 | 1.225000 | 1.225000 | 1.225000 | 1.225000 |
| 1.106250 | 1.106250 | 1.106250 | 1.106250 | 1.106250 | 1.106250 | 1.106250 |
| 1.150000 | 1.150000 | 1.150000 | 1.150000 | 1.150000 | 1.150000 | 1.150000 |
| 1.504544 | 1.492424 | 1.492424 | 1.492424 | 1.492424 | 1.492424 | 1.492424 |
| 1.545454 | 1.545454 | 1.545454 | 1.545454 | 1.545455 | 1.545454 | 1.545454 |
| 1.450000 | 1.450000 | 1.450000 | 1.450000 | 1.450000 | 1.450000 | 1.450000 |
| 1.843332 | 1.899999 | 1.891666 | 1.916665 | 1.891666 | 1.891666 | 1.891666 |
| 2.500000 | 2.500000 | 2.500000 | 2.500000 | 2.500000 | 2.500000 | 2.500000 |
| 1.208333 | 1.191667 | 1.191667 | 1.191667 | 1.191667 | 1.191667 | 1.191667 |
| 1.250000 | 1.250000 | 1.250000 | 1.240000 | 1.250000 | 1.250000 | 1.250000 |
| 1.784847 | 1.772727 | 1.775757 | 1.775757 | 1.775757 | 1.775757 | 1.775757 |
| 1.872726 | 1.872726 | 1.827826 | 1.872726 | 1.872726 | 1.872726 | 1.872726 |
| 1.700000 | 1.666666 | 1.666666 | 1.666666 | 1.666666 | 1.666666 | 1.666666 |
| 2.051666 | 2.041666 | 2.083333 | 2.083333 | 2.083333 | 2.083333 | 2.083333 |
| 2.875000 | 2.875000 | 2.875000 | 2.875000 | 2.875000 | 2.875000 | 2.875000 |

Table A.5. The payoff matrices for the 1974-75 marketing season (\$/bushel of grain) Iowa

|  |  | ${ }^{\theta} 1$ | $\theta_{2}$ | $\begin{gathered} \theta_{3} \\ \text { CORN } \end{gathered}$ | $\theta_{4}$ | $\theta_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (Dec.) | 5.1001 | 3.8464 | 3.3495 | 2.8526 | 1.5989 |
|  | (Jan.) | 4.5531 | 3.7205 | 3.3905 | 3.0605 | 2.2280 |
|  | (Feb.) | 4.7348 | 3.8011 | 3.4310 | 3.0609 | 2.1272 |
|  | (Mar.) | 4.7009 | 3.8164 | 3.4658 | 3.1152 | 2.2306 |
|  | (Apr.) | 4.4715 | 3.7757 | 3.5000 | 3.2242 | 2.5285 |
|  | (May) | 4.5622 | 3.8261 | 3.5343 | 3.2425 | 2.5064 |
|  | (June) | 4.7380 | 3.9006 | 3.5687 | 3.2368 | 2.3995 |
|  | (July) | 5.0636 | 4.0178 | 3.6033 | 3.1888 | 2.1430 |
|  | (Aug.) | 5.6422 | 4.2070 | 3.6380 | 3.0690 | 1.6335 |
| $\mathrm{a}_{10}$ | (Sept.) | 5.4512 | 4.1776 | 3.6728 | 3.1680 | 1.8944 |
|  |  | SOYBEANS |  |  |  |  |
|  | (Nov.) | 12.1168 | 9.3881 | 8.3066 | 7.2251 | 4.4964 |
|  | (Dec.) | 11.9833 | 9.4025 | 8.3795 | 7.3566 | 4.7758 |
|  | (Jan.) | 11.1509 | 9.2181 | 8.4521 | 7.6860 | 5.7533 |
|  | (Feb.) | 11.7205 | 9.4272 | 8.5182 | 7.6091 | 5.3158 |
|  | (Mar.) | 11.9392 | 9.5368 | 8.5845 | 7.6323 | 5.2299 |
|  | (Apr.) | 11.6873 | 9.5127 | 8.6508 | 7.7890 | 5.6146 |
|  | (Mar.) | 13.2962 | 10.0172 | 8.7175 | 7.4178 | 4.1387 |
|  | (June) | 14.7692 | 10.4832 | 8.7844 | 7.0856 | 2.7997 |
|  | (July) | 12.3772 | 9.8524 | 8.8516 | 7.8509 | 5.3261 |
| $\mathrm{a}_{10}$ | (Aug.) | 14.7373 | 10.5706 | 8.9191 | 7.2676 | 3.1009 |

Table A.6. The payoff matrices for the 1975-76 marketing season (\$/bushel of grain) Iowa

|  |  | $\theta_{1}$ | $\theta_{2}$ | $\theta_{3}$ | $\theta_{4}$ | $\theta_{5}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $a_{1}$ (Dec.) | 4.0519 | 2.8540 | $\frac{\text { CORN }}{2.3791}$ | 1.9043 | 0.7063 |  |
| $a_{2}$ (Jan.) | 4.0763 | 2.8896 | 2.4192 | 1.9488 | 0.7621 |  |
| $a_{3}$ (Feb.) | 4.0764 | 2.9184 | 2.4594 | 2.0004 | 0.8424 |  |
| $a_{4}$ (Mar.) | 3.9625 | 2.9113 | 2.4947 | 2.0780 | 1.0268 |  |
| $a_{5}$ (Apr.) | 3.8602 | 2.9071 | 2.5293 | 2.1515 | 1.1983 |  |
| $a_{6}$ (May) | 3.8947 | 2.9417 | 2.5640 | 2.1862 | 1.2333 |  |
| $a_{7}$ (June) | 3.9175 | 2.9734 | 2.5992 | 2.2250 | 1.2810 |  |
| $a_{8}$ (July) | 4.1845 | 3.0747 | 2.6349 | 2.1950 | 1.0852 |  |
| $a_{9}$ (Aug.) | 4.6891 | 3.2434 | 2.6703 | 2.0973 | 0.6515 |  |
| $a_{10}$ (Spet.) | 4.5880 | 3.2402 | 2.7059 | 2.1717 | 0.8238 |  |


| $a_{1}$ (Nov.) | 8.4344 | 5.9642 | 4.9851 | 4.0060 | 1.5357 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $a_{2}$ (Dec.) | 8.1621 | 5.9301 | 5.0454 | 4.1607 | 1.9286 |
| $a_{3}$ (Jan.) | 8.1821 | 5.9791 | 5.1059 | 4.2328 | 2.0299 |
| $a_{4}$ (Feb.) | 8.1391 | 6.0059 | 5.1604 | 4.3149 | 2.1818 |
| $a_{5}$ (Mar.) | 8.1361 | 6.0443 | 5.2151 | 4.3860 | 2.2942 |
| $a_{6}$ (Apr.) | 8.0852 | 6.0691 | 5.2700 | 4.4709 | 2.4148 |
| $a_{7}$ (May) | 9.4180 | 6.4872 | 5.3255 | 4.1639 | 1.2330 |
| $a_{8}$ (June) | 10.8755 | 6.9408 | 5.3813 | 3.8217 | 0.0 |
| $a_{9}$ (July) | 8.4867 | 6.3029 | 5.4373 | 4.5717 | 2.3878 |
| $a_{10}$ (Aug.) | 10.6244 | 6.9500 | 5.4935 | 4.0371 | 0.7252 |

Table A.7. Prediction posterior probability matrices, TPM model, prior probability vector "I"


| 0.6316 | 0. 3634 | 0.0000 | 0 | 0.0000 |
| :---: | :---: | :---: | :---: | :---: |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |
| c. 0000 | 0.0000 | 0.3918 | 0.6082 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.1708 | 0.2457 | 0.5836 |
| 0.0000 | 0. 2308 | 0.3768 | 0.0000 | 0.3925 |
| $0 \cdot 2609$ | 0.0000 | 0.0000 | 0.3913 | $0 \cdot 3478$ |
| 0.3685 | 0.0000 | 0.0000 | 0.0000 | 0.6315 |
| 0.3432 | 0.0000 | 0.0000 | 0.2402 | 0.4166 |
| 0.1464 | 0.5122 | 0.00 | $0 \cdot 3414$ | 0. |
| 0.3896 | 0.2598 | 0.1948 | $0 \cdot 1558$ | 0.0000 |
| 0.0000 | 0.1880 | 0.1253 | 0.2593 | 0.4273 |
| 0.0000 | 0.0000 | 0.2043 | 0.5254 | 0.2703 |
| 0.0000 | 0.4800 | 0.0000 | 0.2000 | 0.3200 |
| 0.3376 | 0.0000 | 0.2403 | 0.0000 | 0.4220 |
| 0.0000 | 0.1948 | 0.0000 | 0.2760 | 0.5292 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.5958 | 0.4042 |
| 0.0000 | 0. 0000 | 0.0000 | 0.0000 | 1.0000 |
| 0.0000 | 0.0000 | 0.5454 | 0.4546 | 0.0000 |
| 0.0000 | 0.3007 | 0.3384 | 0.3609 | 0.0000 |
| 0.5513 | $0 \cdot 0000$ | 0.2206 | $0 \cdot 2281$ | 0.0000 |
| 0.4683 | 0.0000 | 0.2602 | $0 \cdot 1338$ | 0.1377 |
| 0.0000 | 0.0000 | 0.4348 | $0 \cdot 2174$ | 0.3478 |
| 0.3626 | 0.4532 | 0.1842 | 0.0000 | 0.0000 |
| 0.0701 | $0 \cdot 2892$ | 0.2468 | 0.2011 | 0.1928 |
| 0.2005 | 0.4010 | 0.1478 | 0.2506 | 0.0000 |
| 0.0000 | 0.0000 | 0.4655 | 0.3185 | 0.2160 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |


|  | 0.7742 | $0 \cdot 2258$ | 0.0000 | C.OOCO | 0.0000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0009 |
|  | 0.0000 | $0 \cdot 3809$ | $0 \cdot 2381$ | 0.3809 | $0 \cdot 0000$ |
|  | 0.3001 | 0.0529 | 0.1969 | c.onco | 0.4501 |
|  | 0.2430 | C.0000 | 0.2711 | c. 0000 | C.4859 |
|  | 0.4839 | 0.0000 | 0.0000 | 0.3225 | C. 1935 |
|  | 0.0000 | 0.0000 | $0 \cdot 3028$ | C. 3785 | $0 \cdot 3187$ |
|  | 0.0000 | 0.0000 | 0.0452 | 0.5648 | 0.39 CO |
|  | 0.0000 | 0.0000 | 0.3846 | c. 6154 | 0.0000 |
|  | 0.0000 | 0.3704 | $0 \cdot 0000$ | 0.1851 | C. 4444 |
| $\mathrm{Z}_{2}$ |  |  |  |  |  |
|  | 0. 0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 0.0000 | 1.0000 | 0.0000 | C. 0000 | 0.0000 |
|  | 0.3884 | 0.2330 | 0.1456 | 0.2330 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.0000 | 1.0000 | c.0000 |
|  | 0.0000 | 0.0000 | 0.4027 | 0.5973 | 0.000 c |
|  | 0.0000 | 1.0000 | 0.0000 | 0.0000 | $0 \cdot 0000$ |
|  | 0.5236 | 0.0000 | 0.0000 | 0.1091 | 0.3674 |
|  | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |
|  | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 0.0000 | 0.3077 | 0.6923 | 0.0000 | c. 0000 |
| $\mathrm{Z}_{3}$ |  |  |  |  |  |
|  | 0.0790 | 0.2763 | 0.5526 | 0.0921 | 0.0000 |
|  | $0 \cdot 2174$ | 0.4348 | 0.0000 | $0 \cdot 3478$ | C. 0000 |
|  | 0.0000 | C. 1839 | 0.1724 | 0.1839 | C. 4598 |
|  | 0.0000 | 0.2356 | 0.2243 | 0.2898 | C. 2503 |
|  | $0 \cdot 1194$ | 0.2387 | $0 \cdot 2221$ | c. 1811 | C. 2387 |
|  | 0.2575 | 0.0000 | $0 \cdot 2575$ | 0.2790 | c. 2060 |
|  | 0. 1738 | 0.0000 | 0.3259 | 0.2716 | 0.2 .287 |
|  | 0.2614 | 0.3486 | C. 1257 | 0.0961 | 0.1743 |
|  | 0.2598 | 0.1558 | 0.0000 | 0.0000 | 0.5844 |
|  | $0 \cdot 3158$ | 0.5262 | 0.0000 |  | 0.0000 |

Table A. 7 (Continued)



Table A.8. Prediction posterior probability matrices, TPM model, prior probability vector "O"


Table A. 8 (Continued)


Table A.9. Prediction posterior probability matrices, TPM model, prior probability vector "N"

|  |  |  |  |  | $\mathrm{Z}_{1}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.4615 | 0.5385 | 0.0000 | 0.0000 | $0 \cdot 0000$ |  | $0 \cdot 6316$ | $0 \cdot 3684$ | 0.0000 | 0.0000 | 0.0000 |  |
| $0 \cdot 0000$ | 0.0000 | 1.0000 | 0.0000 | 0.0000 |  | 1.0000 | 0.0000 | $0 \cdot 0000$ | 0.0000 | C.OOOC |  |
| 0.0000 | 0.0000 | 0.5630 | $0 \cdot 4370$ | 0.0000 |  | 0.0000 | $0 \cdot 3077$ | 0.3846 | $0 \cdot 3077$ | C.0000 |  |
| 0.0000 | $0 \cdot 0000$ | 0.0000 | 0.0000 | $0 \cdot 0000$ |  | $0 \cdot 1826$ | 0.0644 | C.4.791 | $0 \cdot 0000$ | $0 \cdot 2739$ |  |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.1340 | 0.0000 | 0.5981 | 0.0000 | C. 2679 |  |
| 0.0000 | 0.0000 | 0.3885 | 0.2795 | $0 \cdot 3320$ |  | 0.3659 | 0.0000 | 0.0000 | 0.4878 | C. 1464 |  |
| 0.0000 | 0.1955 | 0.6383 | 0.0000 | $0 \cdot 1662$ |  | 0.0000 | 0.0000 | $0 \cdot 5296$ | $0 \cdot 3 \leq 10$ | $0 \cdot 1393$ |  |
| $0.1875$ | $0.0000$ | $0.0000$ | 0.5625 | $0 \cdot 2500$ |  | $0.0000$ | $0.0000$ | $0.1063$ | 0. 6644 | 0.2294 |  |
| $0.3685$ | $0.0000$ | $0.0000$ | $0.0000$ | $0.6315$ |  | $0.0000$ | $0.0000$ | $0.5556$ | 0.4444 | $0 \cdot 0000$ |  |
| $0 \cdot 2767$ | 0.0000 | 0.0000 | 0.3874 | 0.3359 |  | 0.0000 | 0.4763 | 0.0000 | 0.2380 | C.285? |  |
|  |  |  |  |  | $\mathrm{Z}_{2}$ |  |  |  |  |  |  |
| 0.0790 | $0 \cdot 5526$ | 0.0000 | 0.3684 | $0 \cdot 0000$ |  | $0 \cdot 0000$ | 1.0000 | $0 \cdot 0000$ | 0.0000 | C-0000 | - |
| $0 \cdot 1948$ | 0. 2598 | 0.3896 | $0 \cdot 1558$ | 0.0000 |  | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | $\infty$ |
| 0.0000 | 0.2063 | $0 \cdot 2749$ | 0.2844 | $0 \cdot 2344$ |  | $0 \cdot 2041$ | 0.2449 | $0 \cdot 3061$ | $0 \cdot 2449$ | $0 \cdot 0000$ | $\omega$ |
| 0.0000 | 0.0000 | $0 \cdot 3822$ | 0.4914 | 0.1264 |  | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |  |
| 0.0000 | $0 \cdot 5714$ | $0 \cdot 0000$ | $0 \cdot 2381$ | $0 \cdot 1905$ |  | 0.0000 | 0.0000 | 0.5742 | 0.4258 | C.0000 |  |
| 0.1962 | 0.0000 | $0 \cdot 5586$ | 0.0000 | 0.2452 |  | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |  |
| 0.0000 | $0 \cdot 2648$ | 0.0000 | 0.3754 | $0 \cdot 3598$ |  | 0.4721 | 0.0000 | 0.0000 | 0.1967 | 0.2312 |  |
| $0.0000$ | $0.0000$ | $0.0000$ | 0.0000 | 1.0000 |  | $0.0000$ | $0.0000$ | $1.0000$ | $0.0 C O C$ |  |  |
| $0.0000$ $0.0000$ | $\begin{aligned} & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.7467 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.2533 \\ & 1.0000 \end{aligned}$ |  | $\begin{aligned} & 0 . C 000 \\ & \text { C. OOOO } \end{aligned}$ | $\begin{aligned} & 1.0000 \\ & 0.1818 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.8182 \end{aligned}$ | $\begin{aligned} & 0.00 C O \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 . O C O Q \\ & 0.0000 \end{aligned}$ |  |
|  |  |  |  |  | 73 |  |  |  |  |  |  |
|  | 0.0000 | $0 \cdot 7058$ |  | $0.0000$ |  | $0 \cdot 0261$ | $0 \cdot 1826$ | $0 \cdot 7304$ | 0.0608 | C.0000 |  |
| 0.0000 | $0 \cdot 2247$ | 0.5056 | 0.2697 | 0.0000 |  | $0 \cdot 1219$ | 0.4878 | 0.0000 | 0.3902 | C.0000 |  |
| $0 \cdot 2917$ | $0 \cdot 0000$ | 0.4669 | $0 \cdot 2414$ | $0 \cdot 0000$ |  | 0.0000 | $0 \cdot 1951$ | 0.3653 | 0.1951 | C. 2439 |  |
| 0. 2446 | 0.0000 | $0 \cdot 5436$ | 0.1398 | $0 \cdot 0719$ |  | 0.0000 | 0.2144 | $0 \cdot 4081$ | $0 \cdot 2637$ | C-1139 |  |
| 0.0000 | 0.0000 | $0 \cdot 6897$ | $0 \cdot 1724$ | $0 \cdot 1379$ |  | 0.0572 | 0.2289 | $0 \cdot 4258$ | $0 \cdot 1736$ | O-1144 |  |
| 0.1808 | 0.4519 | 0.3673 | 0.0000 | $0 \cdot 0000$ |  | $0 \cdot 1255$ | 0.0000 | $0 \cdot 5021$ | $0 \cdot 2720$ | 0. 1004 |  |
| 0.0314 | $0 \cdot 2593$ | 0.4426 | 0.1803 | 0.0864 |  | 0.0773 | 000000 | 0.5796 | 0.2415 | $0 \cdot 1017$ |  |
| 0.0957 | $0 \cdot 3828$ | $0 \cdot 2822$ | $0 \cdot 2393$ | 0.0000 |  | $0 \cdot 1440$ | 0.3839 | $0.2769$ | $0.0993$ | $0.0960$ |  |
| $0 \cdot 0000$ | 0.0000 | $0 \cdot 6858$ | $0 \cdot 2346$ | $0 \cdot 0796$ |  | C-2247 | 0.2696 | $0.0000$ | $0.0000$ | $0 \cdot 5056$ |  |
| $0 \cdot 0000$ | $0 \cdot 0000$ | 0.0000 | 0.0000 | 1-0000 |  | $0 \cdot 1875$ | 0.6249 | 0.0000 | 0.1875 | 0.0000 |  |

Table A. 9 (Continued)
$-\boldsymbol{\sim}$

|  |  |  |  |  | $\mathrm{Z}_{4}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0000 | 0.0000 | 1.0000 | C.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0090 |
| 0.0000 | 0.3390 | 0.2542 | 0.4068 | 0.0000 |  | 0.0000 | 0.0000 | $0 \cdot 7595$ | $0 \cdot 1772$ | 0.0633 |
| $0 \cdot 0000$ | $0 \cdot 3716$ | 0.3095 | 0.1923 | $0 \cdot 1266$ |  | 0. 2597 | 0.1558 | $0 \cdot 5844$ | 0.0000 | C.000? |
| 0.0000 | 0.5064 | 0.0000 | $0 \cdot 4341$ | 0.0595 |  | $0 \cdot 2731$ | 0. 3855 | $0 \cdot 3414$ | 0.0000 | C.0090 |
| 0.0000 | 0.5333 | 0.3333 | 0.0000 | $0 \cdot 1333$ |  | 0.1457 | 0.5829 | 0.2714 | 0.0000 | C.0000 |
| $0 \cdot 0919$ | $0 \cdot 4596$ | 0.4484 | 0.0000 | 0.0000 |  | C. 0000 | $0 \cdot 3077$ | 0.6154 | 0.0769 | 0.0050 |
| $0 \cdot 3729$ | $0 \cdot 1206$ | $0 \cdot 3280$ | 0.1785 | 0.0000 |  | 0.3040 | 0.0000 | 0.4559 | 0.0000 | C. 2401 |
| $0 \cdot 1964$ | 0.0000 | $0 \cdot 1819$ | 0.4909 | $0 \cdot 1309$ |  | $0 \cdot 1099$ | 0.0000 | $0 \cdot 5918$ | $0 \cdot 1517$ | C. 1466 |
| 0.0670 | $0 \cdot 2681$ | 0.6600 | 0.0000 | 0.0048 |  | $0 \cdot 1064$ | $0 \cdot 1277$ | 0.6383 | $0 \cdot 1277$ | C.000? |
| $0 \cdot 1994$ | $0 \cdot 0000$ | 0.4117 | $0 \cdot 3390$ | $0 \cdot 0499$ |  | 0.0000 | 0.0000 | 0.4546 | 0.4546 | 0. 0909 |
|  |  |  |  |  | Z |  |  |  |  |  |
| 0.0000 | 0.0000 | 0.4898 | 0.2041 | 0.3061 | 5 | C. 0000 | 0.0000 | C.OCOO | 0.2500 | 0.7500 |
| 0.2500 | 0.0000 | 0.2500 | 0.0000 | $0 \cdot 5000$ |  | 0.0000 | 0.0000 | 0.0000 | $0 \cdot 2857$ | 0.7143 |
| $0 \cdot 0000$ | $0 \cdot 3988$ | 0.4652 | 0.0000 | $0 \cdot 1360$ |  | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.000\% |
| 0.0000 | $0 \cdot 3604$ | 0.4805 | 0.0000 | $0 \cdot 1591$ |  | 0.0000 | $0 \cdot 0000$ | 0.5428 | $0 \cdot 4572$ | $0 \cdot 0000$ |
| $0 \cdot 0000$ | 0.0000 | $0 \cdot 6000$ | $0 \cdot 4000$ | $0 \cdot 0000$ |  | 0.2320 | 0.0000 | $0 \cdot C 000$ | 0.7680 | 0.0000 |
| 0.0000 | 0.0000 | 0.2360 | 0.7640 | $0 \cdot 0000$ |  | 0.0000 | 0.4545 | 0.0000 | 0.0000 | C. 5455 |
| 0.0000 | 0.1656 | 0.5407 | $0 \cdot 2938$ | 0.0000 |  | 0.0930 | 0.0000 | 0.5582 | $0 \cdot 3489$ | C.0000 |
| $0 \cdot 0562$ | 0. 2248 | 0.6815 | 0.0000 | $0 \cdot 0375$ |  | C. 1231 | $0 \cdot 3283$ | $0 \cdot 3787$ | $0 \cdot 1698$ | C.0000 |
| 0.1546 | 0.3092 | 0.0951 | 0.3581 | 0.0829 |  | 0.1198 | 0.1437 | $0 \cdot 3593$ | $0 \cdot 2874$ | 0.0898 |
| 0.0406 | $0 \cdot 3246$ | 0.4817 | 0.1298 | $0 \cdot 0232$ |  | 0.1961 | 0.0000 | 0.5882 | 0.0981 | $0 \cdot 1176$ |

Table A.l0. Prediction posterior probability matrices, TPM model, prior probability vector "P"


|  |  |  |  |  | ${ }^{7} 1$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.4615 | 0.5385 | 0.0000 | 0.0000 | 0.0000 |  | 0.6316 | $0 \cdot 3684$ | 0.0000 | 0.0000 | C.0000 |  |
| $0 \cdot 0000$ | 0.0000 | 1.0000 | 0.0000 | 0.0000 |  | 1.0000 | $0 \cdot 0000$ | $0 \cdot 0000$ | 0.0000 | C.000? |  |
| 0.0000 | 0.0000 | 0.2690 | 0.7310 | 0.0000 |  | 0.0000 | $0 \cdot 1739$ | 0-2174 | 0.6087 | 0.0000 |  |
| 0.0000 | $0 \cdot 0000$ | 0.0000 | 0.0000 | 0.0000 |  | C.0771 | 0.0272 | 0.2022 | $0 \cdot 00 \mathrm{CC}$ | 0.69こ5 |  |
| 0.0000 | 0.0000 | 0.0000 | $0 \cdot 0000$ | 0.0000 |  | 0.0573 | C-0000 | $0 \cdot 2556$ | $0 \cdot 0000$ | $0 \cdot 6371$ |  |
| 0.0000 | 0.0000 | 0.1157 | $0 \cdot 2913$ | $0 \cdot 5930$ |  | $0 \cdot 1240$ | $0 \cdot 0000$ | 0.0000 | 0.5785 | C-29?5 |  |
| 0.0000 | 0. 1068 | $0 \cdot 3486$ | 0.0000 | $0 \cdot 5447$ |  | 0.0000 | $0 \cdot 0000$ | $0 \cdot 2098$ | $0 \cdot 4590$ | $0 \cdot 3312$ |  |
| 0.0513 | $0 \cdot 0000$ | 0.0000 | 0.5385 | 0.4102 |  | 0.0000 | 0.0000 | 0.0279 | $0 \cdot 6107$ | C. 3614 |  |
| 0.0886 | 0. OCOO | 0.0000 | $0 \cdot 0000$ | 0.9114 |  | $0 \cdot 0000$ | $0 \cdot 0000$ | 0.2632 | $0 \cdot 7368$ | $0 \cdot 0000$ |  |
| $0 \cdot 0759$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 3717$ | 0.5525 |  | 0.0000 | 0.1575 | $0 \cdot 00<0$ | $0 \cdot 2755$ | $0 \cdot 5670$ |  |
|  |  |  |  |  | $\mathrm{Z}_{2}$ |  |  |  |  |  |  |
| $0 \cdot 0412$ | 0. 2877 | 0.0000 | $0 \cdot 6712$ | $0 \cdot 0000$ |  | $0 \cdot 0000$ | 1-0000 | 0.0000 | $0 \cdot 0000$ | C-00CO | 1 |
| $0 \cdot 1402$ | $0 \cdot 1870$ | 0.2804 | $0 \cdot 3925$ | $0 \cdot 0000$ |  | 0.0000 | 1-00CC | $0 \cdot 0000$ | 0.0000 | $0 \cdot 0000$ | $\infty$ |
| 0.0000 | $0 \cdot 0716$ | 0.0954 | $0 \cdot 3453$ | $0 \cdot 4878$ |  | 0-1266 | $0 \cdot 1519$ | $0 \cdot 1899$ | $0 \cdot 5316$ | C.0000 | G |
| 0.0000 | 0.0000 | $0 \cdot 1336$ | $0 \cdot 6012$ | 0.2652 |  | 0.0000 | $0 \cdot 0000$ | 0.0000 | 1-0000 | $0 \cdot 0000$ |  |
| 0.0000 | $0 \cdot 2243$ | 0.0000 | $0 \cdot 3271$ | $0 \cdot 4486$ |  | 0.0000 | $0 \cdot 0000$ | $0 \cdot 2781$ | 0.7219 | C.0000 |  |
| $0 \cdot 0881$ | 0.0000 | 0.2509 | 0.0000 | $0 \cdot 6610$ |  | $0 \cdot 0000$ | $1 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | O-0000 |  |
| 0.0000 | 0.0709 | 0.0000 | $0 \cdot 3515$ | 0. 5776 |  | 0.1500 | $0 \cdot 0000$ | 0.0000 | $0 \cdot 2187$ | $C \cdot 6313$ |  |
| 0.0000 | 0.0000 | 0.0000 | $0 \cdot 0000$ | 1-0000 |  | $0 \cdot 0000$ | $0 \cdot 0000$ | 1.0000 | $0 \cdot 0000$ | C-0000 |  |
| $0 \cdot 0000$ | 0.0000 | 0.0000 | $0 \cdot 6323$ | $0 \cdot 3677$ |  | 0.0000 | 1.0000 | 0.000 C | $0 \cdot 0000$ | $0.0 \mathrm{Cc}$ |  |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |  | 0.0000 | $0 \cdot 1818$ | $0 \cdot 8182$ | $0 \cdot 0000$ | $0.0000$ |  |
|  |  |  |  |  | $Z_{3}$ |  |  |  |  |  |  |
| 0. 0000 | $0 \cdot 0000$ | 0.4067 | 0.5933 | $0 \cdot 0000$ |  | $0 \cdot 0226$ | $0 \cdot 1585$ | $0.6340$ | $0 \cdot 1848$ | 0.0000 0.0000 |  |
| $0 \cdot 0000$ | $0 \cdot 1342$ | 0.3020 | $0 \cdot 5638$ | $0 \cdot 0000$ |  | 0.0617 | C-2469 | $0 \cdot 0020$ | O-6914 | - 0000 |  |
| $0 \cdot 1819$ | 0.0000 | 0.2912 | $0 \cdot 5269$ | $0 \cdot 0000$ |  | 0.0000 | $0 \cdot 0721$ | $0 \cdot 1351$ | 0.2523 | $0 \cdot 5405$ |  |
| $0 \cdot 1431$ | 0.0000 | 0.3181 | 0. 2862 | $0 \cdot 2526$ |  | $0 \cdot 0000$ | 0.0962 | $0 \cdot 1831$ | $0 \cdot 4141$ | 0.3066 |  |
| 0.0000 | 0.0000 | $0 \cdot 3252$ | $0 \cdot 2846$ | $0 \cdot 3902$ |  | $0 \cdot 0285$ | $0 \cdot 1141$ | 0.2122 | $0 \cdot 3029$ | 0.3422 |  |
| 0.1808 | 0.4519 | $0 \cdot 3673$ | 0.0000 | 0.0000 |  | 0.0575 | 0.0000 | $0 \cdot 2301$ | $0 \cdot 4363$ | $0 \cdot 2761$ |  |
| $0 \cdot 0167$ | $0 \cdot 1377$ | 0.2351 | $0 \cdot 3352$ | 0.2754 |  | 0.0366 | C. 0000 | $0 \cdot 2744$ | $0 \cdot 4002$ | 0.2888 |  |
| 0.0599 | $0 \cdot 2395$ | 0.1766 | $0 \cdot 5240$ | 0.0000 |  | $0 \cdot 0833$ | 0.2222 | $0 \cdot 1602$ | $0 \cdot 2011$ | $0 \cdot 3332$ |  |
| 0.0000 | 0.0000 | $0 \cdot 3456$ | 0.4138 | 0.2406 |  | 0.0637 | $0 \cdot 0764$ | $0 \cdot 0000$ | $0 \cdot 0000$ | 0.8599 |  |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |  | 0.1277 | 0.4255 | 0.0000 | 0.4469 | $0 \cdot 0000$ |  |

Table A. 10 (Continued)


C
$0 \quad R \quad N$

|  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | $0.0 c 00$ | 1.0000 | 0.0000 |
| 0.0000 | $0 \cdot 168 \mathrm{C}$ | 0.1260 | $0 \cdot 7059$ | 0.0000 | $0 \cdot 0000$ | 0.0000 | 0.4316 | 0.3525 | C-2158 |
| 0.0000 | 0.1758 | 0.1464 | 0.3184 | 0.3595 | $0 \cdot 2597$ | C-1558 | $0 \cdot 5844$ | 0.000 | $0 \cdot 0000$ |
| 0.0000 | $0 \cdot 2125$ | 0.0000 | 0.6375 | $0 \cdot 1499$ | C. 2731 | $0 \cdot 3855$ | $0 \cdot 3414$ | $0 \cdot 0000$ | 0 -0C00 |
| 0.0000 | 0.3200 | 0.2000 | 0.0000 | 0.4800 | C. 1457 | $0 \cdot 5829$ | $0 \cdot 2714$ | $0 \cdot 0000$ | C.0000 |
| 0.0919 | 0.4596 | 0.4484 | 0.0000 | 0.0000 | $0 \cdot 0000$ | $0 \cdot 2581$ | $0 \cdot 5161$ | $0 \cdot 22.58$ | C-0000 |
| $0 \cdot 2578$ | $0 \cdot 0834$ | $0 \cdot 2268$ | 0.4320 | 0.0000 | 0.1382 | 0.0000 | $0 \cdot 2072$ | 0.0000 | 0.6546 |
| 0.0681 | 0.0000 | 0.0631 | $0 \cdot 5963$ | $0 \cdot 2725$ | 0.0520 | 0.0000 | $0 \cdot 2802$ | 0.2514 | $0 \cdot 4164$ |
| $0 \cdot 0655$ | $0 \cdot 2619$ | $0 \cdot 6447$ | 0.0000 | $0 \cdot 0280$ | 0.0806 | 0.0968 | $0 \cdot 4839$ | $0 \cdot 3387$ | $0 \cdot 0000$ |
| 0.0951 | 0.0000 | 0.1963 | 0.5659 | $0 \cdot 1427$ | 0.0000 | 0.0000 | $0 \cdot 1754$ | 0.6140 | 0.2105 |
|  |  |  |  |  |  |  |  |  |  |
| 0.0000 | 0.0000 | 0.1611 | $0 \cdot 2349$ | 0.6040 |  |  |  |  | $\begin{aligned} & 0 \cdot 8372 \\ & 0 \cdot 8109 \end{aligned}$ |
| 0.0714 | 0.0000 | 0.0714 | 0.0000 | $0 \cdot 8571$ | $0.0000$ | $0.0000$ | $0.0000$ | $0.1891$ | $0.8109$ |
| 0.0000 | 0-2374 | $0 \cdot 2769$ | 0.0000 | $0 \cdot 4857$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | 1-0000 | $0 \cdot 0000$ |
| 0.0000 | $0 \cdot 2008$ | $0 \cdot 2677$ | 0.0000 | $0 \cdot 5316$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 2533$ | $0 \cdot 7467$ | C-0000 |
| 0.0000 | 0.0000 | $0 \cdot 3000$ | 0.7000 | 0.0000 | 0.0795 | 0.0000 | $0 \cdot 0000$ | $0 \cdot 9205$ | $0 \cdot 0000$ |
| 0.0000 | 0.0000 | 0.0811 | 0.9189 | 0.0000 | 0.0000 | 0.1219 | 0.0000 | $0 \cdot 0000$ | $0 \cdot 8781$ |
| $0 \cdot 0000$ | 0.0955 | $0 \cdot 3117$ | 0.5928 | $0 \cdot 0000$ | $0 \cdot 0497$ | $0 \cdot 0000$ | 0-2981 | $0 \cdot 6522$ | 0.0000 |
| $0 \cdot 0473$ | 0.1893 | $0 \cdot 5739$ | 0.0000 | 0.1894 | $0 \cdot 0864$ | 0.2305 | O-2658 | $0 \cdot 4173$ | 0.0000 |
| 0.0669 | $0 \cdot 1339$ | 0.0412 | $0 \cdot 5426$ | $0 \cdot 2154$ | 0.0552 | $0 \cdot 0663$ | $0 \cdot 1658$ | $0 \cdot 4641$ | $0 \cdot 2486$ |
| 0.0282 | $0 \cdot 2253$ | 0.3344 | 0.3155 | $0 \cdot 0967$ | $0 \cdot 1069$ | $0 \cdot 0000$ | $0 \cdot 3209$ | $0 \cdot 1872$ |  |

Table A.ll. Prediction posterior probability matrices, MAPM model, prior probability vector "I"


Table A.ll (Continued)


C

|  |  |  |  |  | $Z_{4}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.4296 | $0 \cdot 0000$ | 0.5704 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 1.OCOO | C.000c |
| 0.6061 | $0 \cdot 2424$ | $0 \cdot 1515$ | 0.0000 | 0.0000 |  | 0.5556 | 0.0000 | 0.0000 | $0 \cdot 4444$ | C-CCOO |
| $0 \cdot 1458$ | $0 \cdot 4582$ | $0 \cdot 1627$ | $0 \cdot 2333$ | 0.0000 |  | 0.0000 | 0.0000 | $0 \cdot 0000$ | 1.0000 | 0.0000 |
| 0.0000 | 0.5148 | 0.2720 | 0.0668 | 0.1454 |  | 0.0000 | 0.4348 | $0 \cdot 2174$ | $0 \cdot 3478$ | C.0000 |
| 0.0000 | $0 \cdot 3404$ | 0.2199 | 0.0000 | 0.4398 |  | 0.0000 | 0.5311 | $0 \cdot 0000$ | $0 \cdot 4689$ | $0.000 ?$ |
| 1.0000 | $0 \cdot 0000$ | 0.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.5556 | 0.0000 | 0.4444 |
| 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |  | 0.1000 | 0.0000 | 0.4737 | $0 \cdot 5263$ | C.0000 |
| 0.4419 | 0.0000 | $0 \cdot 5581$ | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.3749 | 0.6251 | C. 0000 |
| 0.0000 | 0.0000 | 0.6276 | $0 \cdot 3724$ | 0.0000 |  | 0.4545 | $0 \cdot 5455$ | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.1753 | 0.5870 | $0 \cdot 2377$ | 0.0000 |  | 0.0000 | 0.4255 | 0.3192 | $0 \cdot 0000$ | $0 \cdot 2553$ |
|  |  |  |  |  | $Z_{5}$ |  |  |  |  |  |
| 0.2465 | 0.0000 | 0.1785 | 0.0000 | 0.5751 |  | 0.2222 | $0 \cdot 1945$ | 0.1945 | 0.0000 | $0 \cdot 3889$ |
| 0.3529 | $0 \cdot 3294$ | $0 \cdot 1765$ | 0.1412 | 0.0000 |  | 0.1500 | 0.4500 | 0.0000 | 0.0000 | $0 \cdot 4000$ |
| 0.4149 | $0 \cdot 1863$ | 0.0772 | 0.0948 | 0.2268 |  | 0.1165 | $0 \cdot 2542$ | 0.1398 | $0 \cdot 1398$ | C-3496 |
| $0 \cdot 5092$ | $0 \cdot 0727$ | 0.2690 | 0.0000 | $0 \cdot 1491$ |  | 0.1342 | $0 \cdot 2013$ | $0 \cdot 1007$ | 0.1611 | C.4027 |
| 0.0000 | $0 \cdot 2659$ | 0.2720 | C.1186 | $0 \cdot 3435$ |  | 0. 2649 | $0 \cdot 0795$ | $0 \cdot 1882$ | $0 \cdot 0701$ | 0.3974 |
| $0 \cdot 2868$ | $0 \cdot 2868$ | 0.1522 | 0.2743 | 0.0000 |  | 0.5882 | 0. 2941 | 0.0000 | 0.0000 | C. 1176 |
| 0.2647 | 0.1320 | 0.2458 | $0 \cdot 1117$ | $0 \cdot 2458$ |  | 0.2500 | 0.0000 | 0.0000 | 0.0000 | $0 \cdot 7500$ |
| 0.3005 | 0.0000 | 0.1108 | 0.1878 | $0 \cdot 4008$ |  | 0.0000 | 0.1923 | 0.2308 | 0.0000 | $0 \cdot 5769$ |
| $0 \cdot 3971$ | 0.0000 | 0.1765 | $0 \cdot 1293$ | C. 2971 |  | 0. 1562 | 0.0938 | 0.2344 | 0. 2813 | $0 \cdot 2344$ |
| 0.3584 | $0 \cdot 3072$ | 0.0000 | 0.0809 | $0 \cdot 2535$ |  | $0 \cdot 3448$ | $0 \cdot 1724$ | 0.0000 | $0 \cdot 1724$ | $0 \cdot 3104$ |

Table A.l2. Prediction posterior probability matrices, MAPM model, prior probability vector "O"


Table A. 12 (Continued)


|  |  |  |  |  | $\mathrm{Z}_{4}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5305 | 0.0000 | 0.4695 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |
| 0.6122 | $0 \cdot 2857$ | 0.1020 | 0.0000 | 0.0000 |  | 0.7895 | c. 0000 | 0.0000 | 0.2105 | C. 0000 |
| 0.1682 | 0.6169 | 0.1252 | 0.0897 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |
| 0.0000 | 0.7248 | 0.2188 | 0.0269 | 0.0294 |  | 0.0000 | 0.6604 | 0.1887 | 0.1509 | 0.0000 |
| 0.0000 | 0.6436 | 0.2376 | 0.0000 | 0.1188 |  | $0 \cdot 0000$ | 0.7985 | 0.0000 | 0.2015 | $0 \cdot 0000$ |
| 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.8333 | 0.0000 | 0.1667 |
| 0.0000 | 0.0000 | 0.0000 | 1.000 C | 0.0000 |  | 0.0000 | 0.0000 | 0.6428 | $0 \cdot 3572$ | D. 0000 |
| 0.5429 | 0.0000 | 0.4571 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.5453 | 0.4547 | 0.0000 |
| 0.0000 | 0.0000 | 0.7712 | 0.2288 | 0.0000 |  | 0.4166 | $0 \cdot 5834$ | 0.0000 | 0.0000 | $0 \cdot 0000$ |
| 0.0000 | $0 \cdot 3029$ | 0.5797 | $0 \cdot 1174$ | 0.0000 |  | 0.0000 | 0.6604 | 0.2830 | 0.0000 | 0.0566 |
|  |  |  |  |  | $\mathrm{Z}_{5}$ |  |  |  |  |  |
| 0.5343 | 0.0000 | 0.2579 | 0.0000 | 0.2078 |  | C. 3453 | 0. 3525 | 0.2014 | 0.0000 | 0.1007 |
| $0 \cdot 3913$ | 0.4261 | C. 1304 | 0.0522 | 0.0000 |  | 0.2023 | $0 \cdot 7079$ | 0.0000 | 0.0000 | $0 \cdot 0899$ |
| 0.5509 | $0 \cdot 2886$ | 0.0683 | 0.0420 | 0.0502 |  | 0.1906 | 0.4853 | 0.1525 | 0.0763 | 0.0953 |
| 0.6379 | 0.1063 | 0.2247 | 0.0000 | 0.0311 |  | 0.2409 | $0 \cdot 4217$ | $0 \cdot 1205$ | 0.0964 | $0 \cdot 1205$ |
| 0.0000 | 0.5273 | 0.3082 | 0.0672 | 0.0973 |  | 0.4625 | 0.1619 | 0.2191 | 0.0408 | 0.1156 |
| 0.3522 | 0.4109 | 0.1246 | 0.1123 | 0.0000 |  | 0.6186 | 0.3608 | 0.0000 | 0.0000 | 0.0206 |
| 0.4006 | 0.2330 | 0.2480 | 0.0564 | 0.0620 |  | 0.6666 | 0.0000 | 0.0000 | 0.0000 | $0 \cdot 3334$ |
| 0.5965 | 0.0000 | 0.1467 | $0 \cdot 1243$ | $0 \cdot 1326$ |  | 0.0000 | 0.4729 | 0.3243 | 0.0000 | 0.2027 |
| 0.6538 | 0.0000 | 0.1937 | 0.0709 | 0.0815 |  | 0.2817 | 0.1972 | $0 \cdot 2817$ | 0.1690 | $0 \cdot 0704$ |
| 0.4560 | 0.4560 | 0.0000 | $0 \cdot 0343$ | 0.0538 |  | 0.5263 | $0 \cdot 3070$ | 0.0000 | 0.0877 | c.c789 |

Table A.13. Prediction posterior probability matrices, MAPM model, prior probability vector "N"


Table A.l3 (Continued)


|  |  |  |  |  | $\mathrm{Z}_{4}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1585 | 0.0000 | 0.8415 | 0. 0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |  |
| -0.3571 | -0.2657 | -0.3571 | 0.0000 0.2140 | 0.0000 0.0000 |  | 0.3846 0.0000 | 0.0000 0.0000 | 0.0000 | 10.6154 1.0000 | 0.0000 0.0000 |  |
| 0.0000 | 0.4294 | 0.4537 | 0.0557 | 0.0611 |  | 0.0000 | 0.3571 | 0.3571 | 0.2857 | 0.0000 |  |
| 0.0000 | 0.3404 | 0.4398 | 0.0000 | 0.2199 |  | 0.0000 | 0.5311 | 0.0000 | 0.4689 | 0.0000 |  |
| 1.0000 | $0 \cdot 0000$ | 0.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.8333 | 0.0000 | 0.1667 |  |
| $0 \cdot 0000$ | 0.0000 | 0.0000 | 1.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.6428 | 0.3572 | 0.0000 |  |
| 0.1653 | 0.0000 | 0.8347 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.5453 | 0.4547 | 0. 0000 |  |
| -0.0000 | - 0.0000 | 0.7712 0.7398 | 0.2288 0.1498 | - 0.0000 |  | 0.2941 0.0000 | 0.7059 0.3571 | - 0.0000 | $\begin{aligned} & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.1072 \end{aligned}$ |  |
| 0.1605 | 0.0000 | 0.4649 | 0.0000 | 0.3745 | $\mathrm{z}_{5}$ |  |  |  |  |  |  |
| 0.1765 | 0.3294 | 0.4529 | -. 1412 | 0.0000 |  | - 0.1255 | 0. 0.6207 | 0.4375 | 0.0000 | 0.2188 0.2759 | $\ldots$ |
| 0.2743 | - 2463 | 0.2041 0.5724 | 0.1254 0.0000 | O.1499 |  | 0.0642 | -. 2804 | 0.3084 | 0.1542 | -0.1928 | \% |
| 0.0000 | 0.2417 | 0.4944 | 0.1078 | 0.1561 |  | - 0.1545 | 0.0927 | -0.4391 | -0.0818 | -. 0.2419 |  |
| $0 \cdot 1421$ | 0. 2843 | 0.3017 | 0.2719 | 0.0000 |  | 0.4546 | 0.4546 | 0.0000 | 0.0000 | $0 \cdot 0909$ |  |
| 0.1336 | 0.1332 0.0000 | 0.4963 0.2916 | 0.1128 0.2471 | 0.1241 0.2636 |  | 0.2500 | 0.0000 | 0.0000 | O.00c0 | -.7500 |  |
| 0. 2394 | 0.0000 | -0.4256 | 0.2471 0.1558 | 0.2636 0.1791 |  | -0.070 | 0.2041 0.0902 | - 0.4898 | -0.0007 |  |  |
| 0.2582 | 0.4426 | 0.0000 | 0.1166 | 0.1826 |  | 0. 2564 | 0.2564 | 0.0000 | 0.2564 | 0.2308 |  |

Table A.l4. Prediction posterior probability matrices, MAPM model, prior probability vector "P"


|  |  |  |  |  | $\mathrm{Z}_{1}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0000 | 0.0000 | 0.1506 | 0.8494 | 0.0000 |  | 0.0790 | $0 \cdot 2764$ | 0.0000 | 0.6446 | 0.0000 |
| 0.0000 | 0.0482 | 0.0602 | 0.1687 | 0.7229 |  | 0.0311 | $0 \cdot 1242$ | 0.0000 | $0 \cdot 3478$ | 0.4969 |
| 0.0000 | 0.0000 | $0 \cdot 1584$ | 0.3974 | $0 \cdot 4442$ |  | 0.0000 | $0 \cdot 1047$ | $0 \cdot 3581$ | 0.5372 | 0.0000 |
| 0.0000 | 0.0000 | 0.2050 | 0.0000 | 0.7950 |  | 0.0000 | $0 \cdot 1282$ | 0.5128 | $0 \cdot 3590$ | 0.0000 |
| 0.0000 | 0.0904 | 0.2335 | 0.6762 | 0.0000 |  | 0.0000 | $0 \cdot 2157$ | $0 \cdot 3404$ | $0 \cdot 4439$ | 0.0000 |
| 0.0000 | 0.0000 | 0.2275 | 0.4241 | 0.3484 |  | 0.0000 | 0.0474 | $0 \cdot 2844$ | $0 \cdot 4976$ | $0 \cdot 1706$ |
| $0 \cdot 0197$ | 0.1997 | $0 \cdot 1837$ | 0.5969 | 0.0000 |  | $0 \cdot 2500$ | 0.0000 | $0 \cdot 7500$ | 0.0000 | 0.0000 |
| 0.0111 | $0 \cdot 3560$ | 0.2436 | $0 \cdot 3893$ | 0.0000 |  | 0.0769 | $0 \cdot 2051$ | 0.0000 | 0.7179 | 0.0000 |
| 0.0000 | 0.2633 | 0.3510 | $0 \cdot 3857$ | 0.0000 |  | 0.0000 | 0.2857 | 0.7143 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |  | 0.0000 | 0.0000 | 1.0000 | $0 \cdot 0000$ | 0.0000 |
|  |  |  |  |  | Z |  |  |  |  |  |
| 0.0200 | 0.2582 | $0 \cdot 1157$ | 0.3263 | $0 \cdot 2797$ |  | 0.0411 | 0.0000 | 0. 2877 | 0.6712 | $0 \cdot 0000$ |
| 0.0000 | 0.0656 | $0 \cdot 2459$ | 0.6885 | 0.0000 |  | 0.0296 | 0.0000 | $0 \cdot 4734$ | 0.4970 | $0 \cdot 0000$ |
| 0.0000 | 0.0950 | $0 \cdot 2475$ | 0.3104 | $0 \cdot 3471$ |  | 0.5554 | 0.0000 | 0.4446 | 0.0000 | 0.0000 |
| 0.0000 | $0 \cdot 2037$ | $0 \cdot 1255$ | 0.4621 | $0 \cdot 2086$ |  | 0.2598 | 0.0000 | 0.1948 | $0 \cdot 5454$ | 0.0000 |
| 0.0000 | 0.0262 | $0 \cdot 3693$ | 0.0000 | 0-6045 |  | $0 \cdot 1281$ | 0.0000 | 0.0811 | $0 \cdot 7908$ | 0.0000 |
| 0.0527 | $0 \cdot 2110$ | $0 \cdot 1034$ | 0.0000 | $0 \cdot 6329$ |  | 0.0000 | $0 \cdot 3637$ | 0.0000 | $0 \cdot 6363$ | 0.0000 |
| 0.1034 | 0.0000 | 0.0000 | 0.0000 | 0.8966 |  | 0.0909 | 0.0000 | $0 \cdot 2728$ | 0.6363 | $0 \cdot 0000$ |
| 0.0344 | 0.0000 | $0 \cdot 1159$ | 0.4822 | $0 \cdot 3674$ |  | 0.0829 | 0.0000 | 0.5305 | 0.3867 | 0.0000 |
| $0 \cdot 0000$ | $0 \cdot 2991$ | 0.0000 | $0 \cdot 4383$ | $0 \cdot 2626$ |  | 0.0000 | $0 \cdot 0930$ | 0.2326 | $0 \cdot 3256$ | $0.3488$ |
| 0.0000 | 0.0000 | $0 \cdot 6064$ | 0.3936 | 0.0000 |  | 0.0000 | 0.1600 | 0.0000 | 0.8400 | 0.0000 |
|  |  |  |  |  | Z |  |  |  |  |  |
| 0.6501 | $0 \cdot 3499$ | 0.0000 | 0.0000 | 0.0000 | 3 | 0.0000 | 1.0000 | $0 \cdot 0000$ | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.4167 | 0.5833 | 0.0000 |  | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | $0 \cdot 4438$ | 0.5562 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | $0 \cdot 0000$ | C.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.8417 | 0.1583 |  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0. 1179 | 0.0000 | 0.8821 | $0 \cdot 0000$ |  | 0.0000 | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | 0.0000 |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 0.6604 | $0 \cdot 3396$ |
| 0.0000 | $0 \cdot 2407$ | 0.7593 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.2222 | 0.7778 | 0.0000 |
| 0.0000 | $0 \cdot 0000$ | 0.0000 | $0 \cdot 0000$ | $0 \cdot 0000$ |  | 0.2143 | 0. 2858 | 0. 0000 | 0.4999 | 0.0000 |
| 0.0838 | 0.0000 | 0.0000 | $0 \cdot 3274$ | 0. 5887 |  | $0 \cdot 1031$ | 0.0000 | $0 \cdot 0000$ | 0.4330 | $0 \cdot 4639$ |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |  | 0.0709 | $0 \cdot 0000$ | $0 \cdot 4255$ | $0 \cdot 2483$ | 0.2553 |

Table A. 14 (Continued)

$C \quad O \quad \mathrm{~N}$

|  |  |  |  |  | $\mathrm{Z}_{4}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1585 | 0.000 C | 0.8415 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |  |
| 0.3571 | 0.2857 | 0.3571 | 0.0000 | 0.0000 |  | 0.1515 | 0.0000 | 0.0000 | 0.8485 | 0.0000 |  |
| 0.0436 | 0.2739 | 0.1945 | 0.4880 | 0.000 C |  | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |  |
| 0.0000 | 0.2972 | 0.3141 | 0.1350 | $0 \cdot 2537$ |  | 0.0000 | 0.2083 | 0.2083 | 0.5833 | 0.0000 |  |
| 0.0000 | 0.1621 | 0.2095 | 0.0000 | 0.6284 |  | 0.0000 | $0 \cdot 2445$ | 0.0000 | 0.7555 | 0.0000 |  |
| 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.4546 | 0.0000 | 0.5454 |  |
| 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.3396 | 0.6604 | 0.0000 |  |
| 0.1653 | 0.0000 | 0.8347 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.2552 | 0.7448 | 0.0000 |  |
| 0.0000 | 0.0000 | 0.4906 | 0.5094 | 0.000 c |  | 0.2941 | $0 \cdot 7059$ | 0.0000 | 0.0000 | 0.0000 |  |
| 0.0000 | $0 \cdot 0803$ | 0.5382 | $0 \cdot 3814$ | 0.0000 |  | 0.0000 | 0.2325 | 0.3489 | 0.0000 | 0.4186 |  |
|  |  |  |  |  | $\mathrm{Z}_{5}$ |  |  |  |  |  |  |
| 0.0559 | 0.0000 | 0.1618 | 0.0000 | 0.7823 |  | 0.0597 | 0.1045 | 0.2090 | 0.0000 | 0.6269 |  |
| 0.1304 | 0. 2435 | 0.2609 | $0 \cdot 3652$ | $0 \cdot 0000$ |  | 0.0435 | 0.2609 | 0.0000 | 0.0000 | 0.6957 |  |
| 0.1330 | 0.1194 | 0.0989 | 0.2127 | 0.4360 |  | 0. 0274 | 0.1194 | 0.1313 | 0.2297 | 0.4923 | $\bigcirc$ |
| $0 \cdot 1940$ | $0 \cdot 0554$ | 0.4099 | 0.0000 | $0 \cdot 3407$ |  | 0.0299 | 0.0898 | 0.0898 | $0 \cdot 2515$ | 0.5389 |  |
| 0.0000 | 0.1179 | 0.2412 | 0.1840 | 0.4569 |  | 0.0654 | 0.0392 | $0 \cdot 1858$ | $0 \cdot 1211$ | 0.5885 |  |
| 0.0846 | $0 \cdot 1692$ | 0.1796 | 0.5666 | $0 \cdot 0000$ |  | $0 \cdot 3125$ | $0 \cdot 3125$ | $0 \cdot 0000$ | 0.0000 | 0.3750 |  |
| 0.0702 | 0.0700 | 0.2609 | 0.2075 | 0.3914 |  | 0.0526 | 0.0000 | 0.0000 | 0.0000 | 0.9474 |  |
| 0.0673 | 0.0000 | 0.0993 | 0.2946 | 0.5387 |  | 0.0000 | 0.0806 | 0. 1935 | 0.0000 | $0 \cdot 7258$ |  |
| $0 \cdot 1048$ | 0.0000 | 0.1862 | 0.2387 | 0.4703 |  | 0.0336 | 0.6403 | 0.2013 | 0.4228 | $0 \cdot 3020$ |  |
| 0.1171 | 0.2008 | 0.0000 | $0 \cdot 1851$ | 0.4970 |  | 0.0917 | 0.0917 | 0.0000 | $0 \cdot 3211$ | 0.4954 |  |

$\begin{array}{ll}\text { Table A.l5. } & \text { prediction posterior probability matrices, TVLM model, prior } \\ \text { probability vector "I" }\end{array}$



Table A. 15 (Continued)


|  |  |  |  |  | $\mathrm{Z}_{4}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0812 | 0. 2842 | 0.1137 | 0.3789 | 0.1421 |  | 0.1600 | C. 0000 | $0.00 c 0$ | 0.2800 | C. 5600 |  |
| 0.0000 | 0.3389 | 0.2543 | 0.4068 | 0.0000 |  | 0.4166 | 0.0000 | $0 \cdot 0000$ | 0.5834 | 0.0000 |  |
| 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.3571 | 0.6429 | 0.0000 |  |
| 0.0000 | 0.7447 | 0.1118 | 0.0000 | 0.1436 |  | 0.0000 | 0.0000 | 0.3571 | 0.6429 | 0.0000 |  |
| 0.0000 | 0.0000 | 0.6593 | 0.3407 | 0.0000 |  | $0 \cdot 3708$ | 0.0000 | $0 \cdot 1023$ | 0.1562 | 0.3708 |  |
| 0.0000 | 0.0000 | 0.3333 | 0.0000 | 0.6667 |  | 0.3333 | 0.1111 | $0 \cdot 33$ 3 | $0 \cdot 1111$ | 0.1112 |  |
| 0.3360 | 0.1600 | 0.1680 | 0.1680 | 0.1680 |  | 0.1333 | 0.0000 | $0 \cdot 2.000$ | 0.2667 | 0.4000 |  |
| 0.0000 | $0 \cdot 5624$ | 0.2499 | 0.0000 | 0.1876 |  | 0.0000 | 0.0000 | $0 \cdot 4288$ | 0.1959 | 0.3753 |  |
| 0.0000 | 0.6602 | 0.0000 | 0.0881 | $0 \cdot 2517$ |  | 0.0000 | 0.1818 | 0.0000 | 0.3636 | 0.4546 |  |
|  |  |  |  |  | $\mathrm{Z}_{5}$ |  |  |  |  |  |  |
| 0.0000 | 0.0000 | 0.2553 | 0.4255 | $0 \cdot 3192$ |  | 0. 1600 | $0 \cdot 2800$ | 0.56 CO | 0.0000 | 0.0010 | 0 |
| 0.3046 | 0.1015 | 0.2284 | 0.3655 | 0.0000 |  | 0.1307 | $0 \cdot 22.88$ | 0.4575 | 0.1830 | 0.0000 |  |
| 0.0000 | 0.1057 | 0.0754 | 0.4226 | $0 \cdot 3962$ |  | c. 2850 | 0.1069 | 0.0950 | 0.0855 | 0.4276 |  |
| 0.0000 | 0.0000 | 0.2362 | 0.2577 | 0.5061 |  | 0. 3727 | 0.0000 | C. 1242 | 0.2236 | 0.2795 |  |
| 0.0000 | 0.1994 | 0.0675 | c. 2094 | 0.5236 |  | 0.2022 | 0.0000 | $0 \cdot 2231$ | c. 1703 | 0.4044 |  |
| 0.3478 | 0.3478 | 0.0435 | 0.0870 | 0.1739 |  | 0.5001 | 0.3333 | 0.0000 | 0.0000 | 0.1666 |  |
| $0 \cdot 2333$ | 0.0666 | 0.1167 | 0.2333 | 0.3500 |  | C. 5714 | 0.0000 | 0.0000 | 0.0000 | 0.4286 |  |
| $0 \cdot 3177$ | 0.0000 | 0.0470 | 0.4235 | 0.2118 |  | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  |
| 0.2143 | 0.2143 | c.1428 | 0.1428 | $0 \cdot 2857$ |  | 0.2817 | c. 3380 | 0.0000 | 0.1690 | 0.2113 |  |

Table A.16. Prediction posterior probability matrices, TVLM model, prior
probability vector "O"


|  |  |  |  |  | $\mathrm{Z}_{1}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.3352 | 0.4562 | 0.2086 | 0.0000 | 0.0000 |  | 0.2917 | 0.5950 | 0.010 | 0.1133 | 0.0000 |  |
| 0.0000 | 0.4826 | 0.3103 | 0.0000 | 0.2069 |  | 0.5173 | 0.000 | 0.4022 | 0.0804 | 0.0000 |  |
| 0.0000 | $0 \cdot 3557$ | 0.5807 | 0.0000 | 0.0635 |  | 0.4788 | 0.0000 | C.4255 | 0.0957 | 0.0000 |  |
| 0.0000 | 0.2508 | 0.5733 | 0.1759 | 0.0000 |  | 0.5358 | 0.0000 | 0.3571 | 0.1071 | 0.0000 |  |
| 0.0000 | 0.4782 | 0.4070 | 0.1147 | 0.0000 |  | 0.3208 | $0 \cdot 3742$ | 0.1475 | 0.1576 | 0.0000 |  |
| 0.0000 | $0 \cdot 0000$ | $0 \cdot 5000$ | 0.5000 | 0.0000 |  | 0.0000 | 0.4120 | 0.0000 | 0.4704 | $0 \cdot 1176$ |  |
| 0.0000 | 0.5714 | $0 \cdot 2857$ | 0.1429 | 0.0000 |  | $0 \cdot 3753$ | 0.0000 | $0 \cdot 3748$ | $0 \cdot 2498$ | 0.0000 |  |
| 0.4091 | 0.0000 | 0.5455 | 0.0000 | 0.0454 |  | 0. 2478 | $0 \cdot 3855$ | 0.2517 | 0.1149 | $0 \cdot 0000$ |  |
| 0.0000 | 0.0000 | 0.6250 | C. 3750 | 0.0000 |  | $0 \cdot 0000$ | $0 \cdot 7778$ | 0.0000 | 0.2222 | 0.0000 | $\mapsto$ |
|  |  |  |  |  | $\mathrm{Z}_{2}$ |  |  |  |  |  | $\checkmark$ |
| 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.4235 | 0.0000 | 0.4941 | $0 \cdot 0824$ | 0.0000 |  |
| 0.0000 | $0 \cdot 0000$ | 1.0000 | 0.0000 | 0.0000 |  | $0 \cdot 2850$ | $0 \cdot 5820$ | 0.0000 | $0 \cdot 1330$ | $0 \cdot 0000$ |  |
| $0 \cdot 0000$ | 0.0000 | 1.0000 | 0.0000 | $0 \cdot 0000$ |  | 0.0000 | $0 \cdot 8872$ | $0 \cdot 1128$ | 0.0000 | $0 \cdot 0000$ |  |
| 0.0000 | $0 \cdot 0000$ | 0.7568 | 0.0000 | 0.2432 |  | 0.0000 | 0.8873 | $0 \cdot 1127$ | 0.0000 | $0 \cdot 0000$ |  |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.8639 | $0 \cdot 1361$ | 0.0000 | 0.0000 |  |
| 0.0000 | $0 \cdot 0000$ | $0 \cdot 2502$ | 0. 7498 | 0.0000 |  | 0.0000 | $0 \cdot 0000$ | 1.0000 | 0.0000 | 0.0000 |  |
| 0.0000 | $0 \cdot 2507$ | $0 \cdot 7493$ | 0.0000 | $0 \cdot 0000$ |  | 0. 3157 | $0 \cdot 0000$ | $0 \cdot 4737$ | 0.2105 | $0 \cdot 0000$ |  |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  | $0 \cdot 3034$ | $0 \cdot 4721$ | $0 \cdot 1541$ | 0.0704 | 0.0000 |  |
| 1.0000 | $0 \cdot 0000$ | 0.0000 | 0.0000 | 0.0000 |  | 0.4546 | $0 \cdot 0000$ | $0 \cdot 4545$ | 0.0909 | 0.0000 |  |
|  |  |  |  |  | $\mathrm{Z}_{3}$ |  |  |  |  |  |  |
| $0 \cdot 6374$ | $0 \cdot 0000$ | $0 \cdot 2231$ | 0.0000 | $0 \cdot 1395$ |  | $0 \cdot 1827$ | $0 \cdot 7462$ | $0 \cdot 0000$ | 0.0711 | $0 \cdot 0000$ |  |
| 0.5232 | $0 \cdot 4070$ | 0.0000 | 0.0698 | 0.0000 |  | 0.0000 | 0.7778 | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 2222$ |  |
| 0.6140 | 0. 2865 | 0.0585 | 0.0409 | 0.0000 |  | 0.0000 | $0 \cdot 8140$ | $0 \cdot 0000$ | $0 \cdot 1860$ | $0 \cdot 0000$ |  |
| 0.7727 | 0.1336 | 0.0000 | 0.0937 | 0.0000 |  | 0.0000 | $0 \cdot 0000$ | 0.3308 | $0 \cdot 2974$ | $0 \cdot 3718$ |  |
| 0.0000 | $0 \cdot 8695$ | 0.0000 | $0 \cdot 1305$ | $0 \cdot 0000$ |  | 0.0000 | $0 \cdot 5343$ | $0 \cdot 3371$ | $0 \cdot 1285$ | C.0000 |  |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |  | 0.0000 | $0 \cdot 7777$ | $0 \cdot 0000$ | 0.1112 | $0 \cdot 1111$ |  |
| 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.5713 | $0 \cdot 0000$ | $0 \cdot 2858$ | 0.0000 | $0 \cdot 1429$ |  |
| 0.0000 | C. 8077 | $0 \cdot 1539$ | 0.0000 | 0.0384 |  | 0.2831 | $0 \cdot 4402$ | $0 \cdot 0361$ | $0 \cdot 1148$ | 0.1258 |  |
| 0.4443 | 0.0000 | 0.3950 | 0.1185 | 0.0423 |  | $0 \cdot 3540$ | 0.2478 | $0 \cdot 3540$ | 0.0000 | $0 \cdot 044.3$ |  |

Table A. 16 (Continued)

| $\mathrm{Z}_{4}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $0.5192$ | 0.1187 | 0.1978 | 0.0371 | 0.4615 | 0.0000 | C.0000 | 0.2692 | 0.2692 |
| $0.0000$ | $0.5644$ | 0.2420 | $0 \cdot 1936$ | 0.0000 | 0.6818 | 0.0000 | 0.0000 | 0.3182 | 0.0000 |
| 0.0000 | 1. 0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | $0 \cdot 5263$ | $0 \cdot 4737$ | 0.0000 |
| 0.0000 | 0.8982 | 0.0770 | 0.0000 | 0.0247 | 0.0000 | 0.0000 | $0.52 \in 3$ | $0 \cdot 4737$ | 0.0000 |
| 0.0000 | 0.0000 | $0 \cdot 7947$ | 0.2053 | $0 \cdot 0000$ | 0.6707 | c. 0000 | $0 \cdot 1233$ | 0.0942 | 0.1118 |
| 0.0000 | 0.0000 | 0.6667 | 0.0000 | 0.3333 | 0.4500 | 0.1749 | $0 \cdot 3000$ | 0.0500 | 0.0250 |
| 0.4676 | $0 \cdot 2597$ | 0.1559 | 0.0779 | $0 \cdot 0390$ | 0.3157 | 0.0000 | 0.3158 | 0.2106 | 0.1579 |
| $0.0000$ $0 \text { ono }$ | $0.7683$ | $0.1951$ | 0.0000 | 0.0366 | $0.0000$ | $0.0000$ | $0.6910$ | $0 \cdot 1578$ | 0.1512 |
| $0.0000$ | $0.9153$ | 0.0000 | $0.0349$ | $0.0498$ | $0.0000$ | $0.5185$ | $0.0000$ | 0.2963 | 0.1852 |
| $Z_{5}$ |  |  |  |  |  |  |  |  |  |
| 0.0000 | 0.0000 | 0.4660 | $0 \cdot 3883$ | 0.1456 | 0.1860 | 0.3799 | 0.4341 | 0.0000 | 0.0000 |
| 0.4369 | 0.1698 | 0.2185 | $0 \cdot 1748$ | 0.0000 | 0.1711 | 0.3405 | 0.3995 | 0.0799 | 0.0000 |
| 0.0000 | $0 \cdot 3240$ | 0.1322 | 0.3703 | $0 \cdot 1736$ | 0.4976 | $0 \cdot 2177$ | 0.1106 | 0.0498 | 0.1244 |
| 0.0000 | 0.0000 | $0 \cdot 4805$ | 0.2621 | 0.2574 | 0.6463 | 0.0000 | 0.1436 | 0.1293 | 0.0808 |
| 0.0000 | 0.5351 | 0.1036 | $0 \cdot 1606$ | $0 \cdot 2007$ | 0.4256 | 0.0000 | 0.3130 | 0.1195 | 0.1419 |
| 0.4138 | 0.4828 | 0.0345 | 0.0345 | 0.0345 | 0.5455 | 0.4242 | 0.0000 | 0.0000 | 0.0303 |
| 0.4445 | 0.1481 | 0.1482 | 0.1482 | 0.1111 | 0.8889 | 0.000 C | 0.0000 | 0.0000 | 0.1111 |
| 0.6045 | 0.0000 | 0.0597 | 0.2687 | 0.0672 | 1.0000 | 0.0000 | C. 0000 | 0.0000 | 0.0000 |
| $0 \cdot 3273$ | $0 \cdot 3818$ | 0.1454 | 0.0727 | 0.0727 | 0. 3669 | D. 5138 | 0.0000 | 0.0734 | 0.0459 |

Table A.17. Prediction posterior probability matrices, TVLM model, prior
probability vector " N "

```
C O R N
SO Y B E A NS
```

|  |  |  |  |  | $\mathrm{Z}_{1}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1415 | 0.3302 | 0.5283 | 0.0000 | 0.0000 |  | 0.1465 | 0.5121 | 0.0060 | 0.3414 | 0.0000 |
| 0.0000 | $0 \cdot 2106$ | 0.4737 | 0.0000 | $0 \cdot 3158$ |  | $0 \cdot 1515$ | 0.0000 | 0.7070 | $0 \cdot 1414$ | 0.0000 |
| 0.0000 | $0 \cdot 1363$ | 0.7786 | 0.0000 | $0 \cdot 0852$ |  | $0 \cdot 1328$ | 0.0000 | $0 \cdot 7079$ | $0 \cdot 1593$ | $0 \cdot 0000$ |
| 0.0000 | $0 \cdot 0873$ | 0.6984 | 0.2143 | $0 \cdot 0000$ |  | $0 \cdot 1613$ | 0.0000 | 0.6451 | 0.1935 | 0.0000 |
| 0.0000 | $0 \cdot 2075$ | 0.6182 | $0 \cdot 1743$ | 0.0000 |  | $0 \cdot 1149$ | $0 \cdot 2297$ | $0 \cdot 3169$ | $0 \cdot 3385$ | 0.0000 |
| 0.0000 | 0.0000 | 0.5000 | 0.5000 | 0.0000 |  | 0.0000 | $0 \cdot 1668$ | 0.0060 | 0.6666 | 0.1666 |
| 0.0000 | 0. 2758 | $0 \cdot 4828$ | $0 \cdot 2414$ | $0 \cdot 0 C 00$ |  | 0.0910 | 0.0000 | $0 \cdot 5454$ | $0 \cdot 3636$ | 0.0000 |
| 0.1035 | 0.0000 | $0 \cdot 8276$ | 0.0000 | $0 \cdot 0689$ |  | $0 \cdot 0797$ | $0 \cdot 2126$ | $0 \cdot 4859$ | $0 \cdot 2218$ | $0 \cdot 0000$ |
| 0.0000 | 0.0000 | 0.6250 | $0 \cdot 3750$ | 0.0000 |  | 0.0000 | 0.5000 | 0.0000 | 0.5000 | 0.0000 |
|  |  |  |  |  | $\mathrm{Z}_{2}$ |  |  |  |  |  |
| 1.0000 | $0 \cdot 0000$ | 0.0000 | $0 \cdot 0000$ | $0 \cdot 0000$ |  | $0 \cdot 1091$ | 0.0000 | 0.7636 | 0.1273 | 0.0000 |
| 0.0000 | $0 \cdot 0100$ | 1.0000 | $0 \cdot 0000$ | $0 \cdot 0000$ |  | 0. 1369 | 0.4795 | 0.0000 | $0 \cdot 3836$ | 0.0000 |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | $0 \cdot 0000$ |  | $0 \cdot 0000$ | $0 \cdot 6921$ | $0 \cdot 3079$ | 0.0000 | 0.0000 |
| 0.0000 | $0 \cdot 0000$ | $0 \cdot 7568$ | $0 \cdot 0000$ | $0 \cdot 2432$ |  | 0.0000 | 0.6923 | $0 \cdot 3077$ | 0.0000 | $0 \cdot 0000$ |
| $0 \cdot 0000$ | 0.0000 | 1.0000 | 0.0000 | $0 \cdot 0000$ |  | 0.0000 | 0.6445 | $0 \cdot 3555$ | 0.0000 | $0 \cdot 0000$ |
| 0.0000 | 0.0000 | 0.2502 | $0 \cdot 7498$ | 0.0000 |  | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |
| $0 \cdot 0000$ | $0 \cdot 0873$ | 0.9127 | 0.0000 | 0.0000 |  | $0 \cdot 0714$ | 0.0000 | 0-6429 | 0.2857 | $0 \cdot 0000$ |
| 0.0000 | $0 \cdot 0000$ | 0.0000 | $0 \cdot 0000$ | $0 \cdot 0000$ |  | 0.1234 | $0 \cdot 3291$ | $0 \cdot 3759$ | $0 \cdot 1716$ | $0 \cdot 0000$ |
| 1.0000 | $0 \cdot 0000$ | 0.0000 | 0.0000 | 0.0000 |  | $0 \cdot 1220$ | 0.0000 | $0 \cdot 7317$ | $0 \cdot 1463$ | $0 \cdot 0000$ |
|  |  |  |  |  | $\mathrm{Z}_{3}$ |  |  |  |  |  |
| $0 \cdot 2266$ | 0.0000 | $0 \cdot 4759$ | $0 \cdot 0000$ | $0 \cdot 2975$ |  | $0 \cdot 0967$ | $0 \cdot 6775$ | $0 \cdot 0000$ | 0. 2258 | $0 \cdot 0000$ |
| $0 \cdot 3191$ | $0 \cdot 4256$ | $0 \cdot 10000$ | 0.2553 | $0 \cdot 0000$ |  | $0 \cdot 0000$ | 0.5000 | 0-0000 | $0 \cdot 0000$ | $0 \cdot 5000$ |
| $0 \cdot 3608$ | $0 \cdot 2886$ | 0-2064 | $0 \cdot 1443$ | $0 \cdot 0000$ |  | 0-0000 | $0 \cdot 5556$ | $0 \cdot 0000$ | $0 \cdot 4444$ | $0 \cdot 0000$ |
| $0 \cdot 4942$ | 0.1465 | 0.0000 | $0 \cdot 3594$ | 0.0000 |  | 0.0000 | 0.0000 | $0 \cdot 3308$ | $0 \cdot 2974$ | 0.3718 |
| $0 \cdot 0000$ | 0-6557 | 0.0000 | $0 \cdot 3443$ | $0 \cdot 0000$ |  | 0.0000 | $0 \cdot 2.469$ | $0 \cdot 5452$ | $0 \cdot 2079$ | $0 \cdot 0000$ |
| $0 \cdot 0000$ | $0 \cdot 0000$ | 1-0000 | $0 \cdot 0000$ | $0 \cdot 0000$ |  | 0.0000 | $0 \cdot 4999$ | $0 \cdot 0000$ | $0 \cdot 2502$ | $0 \cdot 2499$ |
| 0.0000 | 1.0000 | 0.0000 | $0 \cdot 0000$ | $0 \cdot 0000$ |  | 0.1818 | 0.0000 | $0 \cdot 5455$ | $0 \cdot 0000$ | 0-2728 |
| 0.0000 | 0. 5454 | $0 \cdot 3637$ | 0.0000 | 0.0909 |  | $0 \cdot 1049$ | $0 \cdot 2797$ | 0.0802 | $0 \cdot 2554$ | $0 \cdot 2797$ |
| 0.1176 | $0 \cdot 0000$ | $0 \cdot 6272$ | $0 \cdot 1881$ | 0.0672 |  | $0 \cdot 1117$ | $0 \cdot 1341$ | $0 \cdot 6704$ | 0.0000 | $0 \cdot 0838$ |

Table A. 17 (Continued)


|  |  |  |  |  | $2_{4}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0405 | $0 \cdot 2836$ | 0.2269 | $0 \cdot 3781$ | $0 \cdot 0709$ |  | $0 \cdot 1250$ | 0.0000 | C. 0000 | $0 \cdot 4375$ | 0.4375 |
| 0.0000 | 0.2702 | 0.4054 | $0 \cdot 3244$ | 0.0000 |  | $0 \cdot 2631$ | 0.0000 | 0.0000 | $0 \cdot 7369$ | 0.0000 |
| 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | $0 \cdot 5263$ | 0.4737 | 0.0000 |
| 0.0000 | $0 \cdot 7160$ | 0.2150 | 0.0000 | $0 \cdot 0690$ |  | 0.0000 | $0 \cdot 0000$ | $0 \cdot 5263$ | $0 \cdot 4737$ | $0 \cdot 0000$ |
| 0.0000 | $0 \cdot 0000$ | $0 \cdot 7947$ | $0 \cdot 2053$ | $0 \cdot 0000$ |  | 0.2534 | 0.0000 | 0.2796 | 0.2.35 | $0 \cdot 2534$ |
| 0.0000 | $0 \cdot 0000$ | 0.6667 | 0.0000 | $0 \cdot 3333$ |  | 0.1500 | 0.1000 | $0 \cdot 6000$ | $0 \cdot 1000$ | $0 \cdot 0500$ |
| 0.1834 | $0 \cdot 1746$ | 0. 3668 | 0.1834 | $0 \cdot 0917$ |  | 0.0714 | 0.0000 | $0 \cdot 4286$ | 0. 2858 | $0 \cdot 2143$ |
| 0.0000 | $0 \cdot 4865$ | 0.4324 | 0.0000 | $0 \cdot 0811$ |  | 0.0000 | 0.0000 | 0.6910 | $0 \cdot 1578$ | $0 \cdot 1512$ |
| 0.0000 | 0.7553 | 0.0000 | 0.1008 | $0 \cdot 1440$ |  | 0.0000 | $0 \cdot 2353$ | 0.0000 | $0 \cdot 4706$ | $0 \cdot 2941$ |
|  |  |  |  |  | $Z_{5}$ |  |  |  |  |  |
| 0.0000 | 0. 0000 | 0.4660 | $0 \cdot 3883$ | C. 1456 |  | 0.0540 | 0.1892 | 0.7568 | $0 \cdot 0000$ | $0 \cdot 0000$ |
| $0 \cdot 1415$ | $0 \cdot 0943$ | $0 \cdot 4245$ | $0 \cdot 3396$ | $0 \cdot 0000$ |  | $0 \cdot 0469$ | $0 \cdot 1643$ | $0 \cdot 6573$ | 0.1315 | $0 \cdot 0000$ |
| 0.0000 | $0 \cdot 8204$ | $0 \cdot 1720$ | $0 \cdot 4818$ | 0-2258 |  | 0-1929 | $0 \cdot 1447$ | $0 \cdot 2572$ | $0 \cdot 1158$ | $0 \cdot 2894$ |
| 0.0000 | 0.0000 | 0.4805 | $0 \cdot 2621$ | $0 \cdot 2574$ |  | 0.2334 | $0 \cdot 0000$ | $0 \cdot 3113$ | 0.2802 | $0 \cdot 1751$ |
| 0.0000 | 0. 2475 | $0 \cdot 1676$ | $0 \cdot 2599$ | $0 \cdot 3249$ |  | 0.1099 | $0 \cdot 0000$ | $0 \cdot 4851$ | 0.1851 | $0 \cdot 2199$ |
| $0 \cdot 2222$ | $0 \cdot 4444$ | $0 \cdot 1111$ | $0 \cdot 1111$ | $0 \cdot 1111$ |  | $0 \cdot 3750$ | $0 \cdot 5000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 1250$ |
| 0.1414 | 0.0808 | 0.2828 | 0.2828 | $0 \cdot 2121$ |  | 0.5714 | 0.0000 | $0 \cdot 0000$ | 0.0000 | $0 \cdot 4286$ |
| $0 \cdot 2030$ | $0 \cdot 0000$ | 0.1203 | 0.5414 | $0 \cdot 1353$ |  | 1.0000 | 0.0000 | 0.0000 | 0.0000 | $0 \cdot 0000$ |
| $0 \cdot 1200$ | 0.2400 | 0.3200 | 0.1600 | $0 \cdot 1600$ |  | 0.1869 | 0.4486 | 0.0000 | $0 \cdot 2243$ | 0.1402 |

Table A.18. Prediction posterior probability matrices, TVLM model, prior probability vector "p"

| $\mathrm{Z}_{1}$ |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1415 | $0 \cdot 3302$ | $0 \cdot 5283$ | 0.0000 | 0.0000 | 0.0790 | $0 \cdot 2763$ | 0.0000 | $0 \cdot 5447$ | 0.0000 |  |
| 0.0000 | $0 \cdot 0817$ | 0.1837 | 0.0000 | $0 \cdot 7347$ | $0 \cdot 1120$ | 0.0000 | $0 \cdot 5224$ | $0 \cdot 3657$ | 0.0000 |  |
| 0.0000 | $0 \cdot 0956$ | 0.5461 | 0.0000 | $0 \cdot 3584$ | 0. 0950 | 0.0000 | 0.5063 | 0.3988 | 0.0000 |  |
| 0.0000 | $0 \cdot 0569$ | 0.4548 | 0.4884 | $0 \cdot 0000$ | 0.1087 | 0.0000 | 0.4348 | 0.4565 | 0.0000 |  |
| 0.0000 | $0 \cdot 1446$ | 0.4306 | $0 \cdot 4248$ | $0 \cdot 0000$ | 0.0622 | $0 \cdot 1244$ | $0 \cdot 1716$ | $0 \cdot 6417$ | 0.0000 |  |
| 0.0000 | 0.0000 | $0 \cdot 2222$ | 0.7778 | 0.0000 | 0.0000 | 0.0477 | 0.0000 | 0.6666 | $0 \cdot 2857$ |  |
| 0.0000 | 0.1720 | $0 \cdot 3011$ | $0 \cdot 5269$ | 0.0000 | $0 \cdot 0477$ | 0.0000 | $0 \cdot 2857$ | $0 \cdot 6666$ | 0.0000 |  |
| 0.0769 | $0.0000$ | $0.6155$ | $0.0000$ | $0.3076$ | $0.0513$ | $0.1367$ | $0 \cdot 3126$ | $0.4994$ | $0.0000$ |  |
| 0.0000 | $0 \cdot 0000$ | $0 \cdot 3226$ | 0.6774 | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 2222$ | $0 \cdot 0000$ | $0 \cdot 7778$ | $0 \cdot 0000$ |  |
| 1-0000 | $0 \cdot 0000$ | 0.0000 | 0.0000 | $0.0000{ }^{Z_{2}}$ | 0.0828 | 0.0000 | C. 5793 | $0 \cdot 3379$ | 0.0000 | $\stackrel{N}{N}$ |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0699 | $0.2448$ | 0.0000 | 0.6853 | 0.0000 |  |
| 0.0000 | $0 \cdot 0000$ | 1.0000 | 0.0000 | $0 \cdot 0000$ | 0.0000 | $0 \cdot 6921$ | $0 \cdot 3079$ | $0 \cdot 0000$ | C.0000 |  |
| $0 \cdot 0000$ | 0-0000 | 0.3415 | $0 \cdot 0000$ | 0.6585 | $0 \cdot 0000$ | 0.6923 | $0 \cdot 3077$ | 0.0000 | 0.0000 |  |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.6445 | $0 \cdot 3555$ | 0.0000 | 0.0000 |  |
| 0.0000 | $0 \cdot 0000$ | 0.0870 | 0.9130 | 0.0000 | $0 \cdot 0000$ | 0.0000 | 1.00c0 | 0.0000 | 0.0000 |  |
| 0.0000 | 0.0873 | 0.9127 | 0.0000 | 0.0000 | 0.0417 | C. 0000 | $0 \cdot 3750$ | $0 \cdot 5833$ | 0.0000 |  |
| 0.0000 | 0.0000 | $0.0000$ | $0.0000$ | $0.0000$ | $0.0863$ | $0.2303$ | $0 \cdot 2630$ | $0.4204$ | $0.0000$ |  |
| 1.0000 | $0 \cdot 000 \mathrm{C}$ | 0.0000 | 0.0000 | 0.0000 | $0 \cdot 0893$ | 0.0000 | $0 \cdot 5357$ | $0 \cdot 3750$ | 0.0000 |  |
| $\mathrm{Z}_{3}$ |  |  |  |  |  |  |  |  |  |  |
| 0.0911 | 0.0000 | 0.1913 | 0.0000 | 0.7175 | 0.0618 | 0.4330 | C.0000 | 0.5052 | 0.0000 |  |
| $0-1948$ | 0. 2598 | 0.0000 | 0.5454 | $0 \cdot 0000$ | 0.0000 | $0 \cdot 1429$ | $0 \cdot 0000$ | 0.0000 | $0 \cdot 8571$ |  |
| 0.2651 | 0.2121 | 0.1516 | $0 \cdot 3712$ | $0 \cdot 0000$ | 0.0000 | $0 \cdot 2632$ | 0.0000 | $0 \cdot 7368$ | $0 \cdot 0000$ |  |
| $0 \cdot 2603$ | $0 \cdot 0772$ | 0.0000 | $0 \cdot 6625$ | $0 \cdot 0000$ | 0.0000 | 0.0000 | $0 \cdot 0918$ | 0.2890 | $0 \cdot 6192$ |  |
| 0.0000 | 0. 3524 | 0.0000 | 0.6476 | 0.0000 | 0.0000 | $0 \cdot 1625$ | $0 \cdot 3587$ | $0 \cdot 4788$ | $0 \cdot 0000$ |  |
| $0 \cdot 0000$ | $0 \cdot 0000$ | 1.0000 | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 0000$ | $0 \cdot 1739$ | $0 \cdot 0000$ | $0 \cdot 3045$ | $0 \cdot 5216$ |  |
| 0.0000 | 1.0000 | 0.0000 | $0 \cdot 0000$ | 0.0000 | 0.0769 | 0.0000 | $0 \cdot 2308$ | 0.0000 | $0 \cdot 6923$ |  |
| $0.0000$ | $0 \cdot 3750$ | 0.2501 | 0.0000 | $0 \cdot 3749$ | 0.0345 | $0 \cdot 0921$ | $0 \cdot 0264$ | $0 \cdot 2943$ | - - 5526 |  |
| 0.0651 | 0.0000 | 0.3473 | $0 \cdot 3645$ | 0.2231 | 0.0787 | C.0945 | 0.4724 | 0.0000 | 0.3543 |  |

Table A. 18 (Continued)


|  |  |  |  |  | $\mathrm{Z}_{4}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0176 | 0.1233 | 0.0986 | $0 \cdot 5755$ | 0.1850 |  | 0.0292 | 0.0000 | 0.0000 | 0.3577 | 0.61 .31 |
| 0.0000 | $0 \cdot 1492$ | $0 \cdot 2239$ | 0.6269 | 0.0000 |  | 0.0926 | 0.0000 | 0.0000 | 0.9074 | 0.0000 |
| 0.0000 | 1-0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | $0 \cdot 2409$ | $0 \cdot 759.1$ | $0 \cdot 0000$ |
| 0.0000 | 0.5323 | 0.1598 | 0.0000 | 0.3079 |  | 0.0000 | 0.0000 | 0.2409 | 0.7591 | 0.0000 |
| 0.0000 | $0 \cdot O C O 0$ | $0 \cdot 5252$ | $0 \cdot 4748$ | $0 \cdot 0000$ |  | $0 \cdot 0905$ | 0.0000 | 0.0998 | 0.2668 | $0 \cdot 5429$ |
| 0.0000 | 0.0000 | 0.2500 | 0.0000 | 0.7500 |  | $0 \cdot 1000$ | 0.0666 | $0 \cdot 4000$ | 0.2332 | $0 \cdot 2002$ |
| 0.0957 | 0.0911 | 0.1913 | $0 \cdot 3349$ | $0 \cdot 2870$ |  | $0 \cdot 0256$ | $0 \cdot 0000$ | $0 \cdot 1538$ | $0 \cdot 3590$ | 0.4615 |
| 0.0000 | $0 \cdot 3451$ | 0.3076 | 0.0000 | $0 \cdot 3464$ |  | 0.0000 | $0 \cdot 0000$ | $0 \cdot 3213$ | 0.2569 | 0.4219 |
| 0.0000 | $0 \cdot 3831$ | 0.0000 | 0.1788 | 0.4381 |  | 0.0000 | 0.0645 | 0.0000 | 0.4516 | 0.4839 |
|  |  |  |  |  | $Z_{5}$ |  |  |  |  |  |
| 0.0000 | 0.0000 | $0 \cdot 1727$ | 0.5036 | 0.3238 |  | $0 \cdot 0540$ | $0 \cdot 1892$ | $0.7568$ | 0.0000 | $0 \cdot 0000$ |
| 0.0765 | $0 \cdot 0510$ | $0 \cdot 2296$ | $0 \cdot 6429$ | 0.0000 |  | 0.0353 | $0 \cdot 1237$ | 0.4947 | $0 \cdot 3463$ | 0.0000 |
| $0 \cdot 0000$ | $0 \cdot 0361$ | 0.0516 | 0.5058 | 0.4065 |  | $0 \cdot 0705$ | $0 \cdot 0529$ | $0 \cdot 0940$ | $0 \cdot 1481$ | $0 \cdot 6346$ |
| 0.0000 | 0.0000 | $0 \cdot 1633$ | $0 \cdot 3118$ | 0. 5249 |  | 0.0906 | 0.0000 | $0 \cdot 1208$ | $0 \cdot 3807$ | C.4079 |
| 0.0000 | 0.0756 | 0.0512 | 0.2778 | 0.5954 |  | 0.0429 | 0.0000 | $0 \cdot 1893$ | $0 \cdot 2529$ | $0 \cdot 5149$ |
| 0.1212 | $0 \cdot 2424$ | 0.0606 | $0 \cdot 2121$ | $0 \cdot 3636$ |  | 0.2308 | $0 \cdot 3077$ | 0.0000 | 0.0000 | 0.4614 |
| 0.0511 | $0 \cdot 0292$ | $0 \cdot 1022$ | $0 \cdot 3577$ | 0.4599 |  | $0 \cdot 1818$ | 0.0000 | 0.0000 | 0.0000 | $0 \cdot 8182$ |
| 0.0670 | 0.0000 | 0.0397 | $0 \cdot 6253$ | 0.2680 |  | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0545 | 0.1091 | $0 \cdot 1454$ | 0.2545 | 0.4364 |  | 0.0826 | 0.1984 | 0.0000 | 0.3471 | $0 \cdot 3719$ |

Table A.19. Prediction posterior probability matrices, SEM model, prior probability vector "I"


Table A. 19 (Continued)


Table A.20. Prediction posterior probability matrices, SEM model, prior probability vector "O"


Table A. 20 (Continued)


Table A. 21. Prediction posterior probability matrices, SEM model, prior


Table A. 21 (Continued)


C $\quad \mathrm{R} \quad \mathrm{N}$


```
Table A.22. Prediction posterior probability matrices, SEM model, prior probability vector "P"
```



|  |  |  |  |  | $\mathrm{Z}_{1}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0990 | 0.3465 | 0.5545 | 0.0000 | 0.0000 |  |  |  |
| 0.0000 | 0.0816 | 0.1837 | 0.0000 | $0 \cdot 7347$ |  |  |  |
| 0.0000 | 0.0956 | 0.5461 | 0.0000 | 0.3584 |  |  |  |
| 0.0000 | 0.0000 | 0.3636 | 0.6364 | $0 \cdot 0000$ |  |  |  |
| 0.0000 | 0.1212 | 0.4546 | 0.4242 | $0 \cdot 0000$ |  |  |  |
| 0.0000 | $0 \cdot 2000$ | 0.8000 | 0.0000 | 0.0000 |  |  |  |
| 0.0000 | 0.1564 | 0.3648 | 0.4789 | 0.0000 |  |  |  |
| 0.0769 | 0.0000 | 0.6154 | 0.0000 | $0 \cdot 3077$ |  |  |  |
| 0.0000 | 0.0000 | 0.4878 | 0.5122 | 0.0000 |  |  |  |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |  |  |  |
|  |  |  |  |  | $\mathrm{Z}_{2}$ |  |  |
| 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  |  | O |
| 0.0000 | 0.0000 | 0.2632 | 0.7368 | 0.0000 |  |  |  |
| 0.0000 | 0.0000 | 0.2759 | 0.0000 | 0.7241 |  |  |  |
| 0.0000 | 0.0000 | $0 \cdot 3414$ | 0.0000 | 0.6586 |  |  |  |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |  |  |  |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |  |  |  |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  |  |  |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  |  |  |
| 0.0997 | 0.0000 | 0.0000 | 0.5585 | 0.3418 |  |  |  |
| 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  |  |  |
|  |  |  |  |  | $\mathrm{Z}_{3}$ |  |  |
| 0.0486 | 0.0000 | 0.2719 | 0.0000 | 0.6796 |  |  |  |
| 0.4286 | 0.5714 | 0.0000 | 0.0000 | 0.0000 |  |  |  |
| 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |  |  |  |
| 0.0000 | 0.2647 | 0.1176 | 0.6276 | 0.0000 |  |  |  |
| 0.0000 | 0.3636 | 0.0000 | 0.6364 | 0.0000 |  |  |  |
| 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |  |  |  |
| 0.0000 | 0. 3913 | 0.6087 | 0.0000 | 0.0000 |  |  |  |
| 0.0000 | 0.0000 | 1.0000 | 0.0 .000 | 0.0000 |  |  |  |
| 0.0838 | 0.0000 | 0.4470 | 0.4692 | 0.0000 |  |  |  |
| 0.0000 | 0.0000 | 0.5384 | 0.0000 | 0.4616 |  |  |  |

Table A. 22 (Continued)

$Z_{4}$

| 0.0326 | 0.1143 | 0.0914 | 0.5332 | 0.2285 |
| :---: | :---: | :---: | :---: | :---: |
| 0.0000 | $0 \cdot 1493$ | 0.2239 | 0.6269 | 0.0000 |
| 0.0000 | 0.1051 | 0.1515 | 0.7424 | 0.0000 |
| 0.0000 | 0.3641 | 0.3237 | 0.0000 | 0.3122 |
| 0.0000 | 0.0000 | 0.5172 | 0.4828 | 0.0000 |
| 0.0000 | 0.0505 | 0.0000 | 0.7071 | $0 \cdot 2424$ |
| 0.1239 | 0.0708 | 0.0000 | 0.4336 | 0.3717 |
| 0.0000 | $0 \cdot 3750$ | 0.2500 | 0.0000 | 0.3750 |
| 0.0000 | 0.2011 | 0.0000 | 0.2816 | $0 \cdot 5173$ |
| 0.0000 | 0.0000 | 0.1446 | 0.6075 | 0.2479 |
| 0.0000 | 0.0000 | 0.1558 | 0.4545 | 0.3897 |
| 0.0765 | 0.0510 | 0.2296 | 0.6428 | 0.0000 |
| 0.1090 | 0.0436 | 0.0623 | 0.4580 | 0.3271 |
| 0.1175 | 0.0000 | 0.1045 | 0.2743 | 0.5037 |
| 0.0000 | 0.0792 | 0.0495 | $0 \cdot 2772$ | 0.5941 |
| 0.0948 | 0.0237 | 0.0948 | $0 \cdot 3317$ | 0.4550 |
| 0.0494 | 0.0282 | 0.1317 | 0.3459 | 0.4447 |
| 0.0698 | 0.0000 | 0.0000 | 0.6512 | 0.2791 |
| 0.0735 | 0.1470 | 0.1959 | 0.2057 | 0.3779 |
| 0.0752 | $0 \cdot 2004$ | 0.1002 | 0.2806 | 0.3436 |

```
Table A.23. Prediction posterior probability matrices, CBT-F model, prior
probability vector "I"
```




Table A.24. Prediction posterior probability matrices, CBT-F model, prior probability vector "O"


Table A.25. Prediction posterior probability matrices, CBT-F model, prior probability vector "N"


Table A. 26. Prediction posterior probability matrices, CBT-F model, prior probability vector "P"


|  |  |  |  |  | $\mathrm{Z}_{1}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5089 | 0.0000 | 0.4911 | 0.0000 | 0.0000 |  | 1.0000 | 0.0000 | 0.0000 | 0.0000 | c.0000 |
| 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 1.0000 | 0.0000 | c. 0000 |
| 0.0000 | 0.0000 | 0.0509 | $0 \cdot 9491$ | $0 \cdot 0000$ |  | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | C.0000 |
|  |  |  |  |  |  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  |  |  |  |  | $\mathrm{Z}_{2}$ |  |  |  |  |  |
| 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |  | 0.1250 | 0.8750 | 0.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.2937 | $0 \cdot 7063$ | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |
| 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | -0.0000 | 1.0000 | 0.00n |
| 0.1111 | 0.4445 | 0.0000 | 0.0000 | 0.4444 |  | 0.0000 | D. 0000 | 0.0000 | 0.0000 | 0.0000 |
|  |  |  |  |  |  | 0.0000 | 0.0000 | 0.0000 | 0.00 CO | 0.0000 |
|  |  |  |  |  | 2 |  |  |  |  |  |
| 0.0000 | 0.5272 | 0.4728 | 0.0000 | 0.0000 |  | 0.0000 | 0.2000 | 0.8000 | 0.0000 | 0.0000 |
| 0.0000 | 0.1426 | 0.2861 | 0.3002 | 0.2712 |  | 0.0510 | $0 \cdot 2296$ | 0.0765 | 0.6429 | 0.0000 |
| $0.0000$ | $0.0838$ | 0.2053 | 0.4593 | 0.2515 |  | $0 \cdot 0224$ | 0.1007 | 0.1919 | 0.2821 | 0.4029 |
| $0.0509$ | $0.0000$ | 0.5424 | 0.0000 | 0.4067 |  | 0.0476 | 0.0000 | $0 \cdot 2857$ | 0.6666 | 0.0000 |
|  |  |  |  |  |  | 0.0000 | 0.0000 | 0.0000 | c.0000 | 1.0000 |
|  |  |  |  |  | $\mathrm{Z}_{4}$ |  |  |  |  |  |
| 0.0000 | 0.0000 | 0.2618 | 0.7382 | 0.0000 |  | 0.0000 | 0.0698 | $0 \cdot 2.791$ | 0.6512 | 0.0000 |
| 0.0000 | 0.0000 | 0.1502 | 0.5696 | 0.2802 |  | 0.0557 | 0.0418 | 0.1671 | 0.2340 | 0.5014 |
| 0.0000 | $0 \cdot 2500$ | 0.0000 | 0.0000 | $0 \cdot 7500$ |  | 0.2077 | 0.0000 | $0 \cdot 3561$ | 0.4362 | 0.0000 |
| 0.0330 | 0.0000 | 0.0879 | 0.6154 | 0.2637 |  | 0.0509 | 0.0000 | 0.2542 | $0.2373$ |  |
|  |  |  |  |  |  | 0.0445 | $0 \cdot 1067$ | 0.2001 | $0.3735$ | $0.2752$ |
|  |  |  |  |  | $\mathrm{Z}_{5}$ |  |  |  |  |  |
| 0.0000 | 0.0134 | 0.0599 | 0.4054 | 0.5213 |  | 0.0000 | 0.0000 |  | 0.2800 |  |
| 0.0000 | 0.2954 | 0.1422 | 0.0000 | 0.5623 |  | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |
| 0.5052 | 0.0000 | 0.4948 | 0.0000 | 0.0000 |  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0476 | 0. 1905 | 0.1269 | 0.4444 | 0.1906 |  | 0.0714 | 0.0000 | 0.1072 | 0.5000 | $0 \cdot 3214$ |
|  |  |  |  |  |  | 0.0950 | 0.1140 | 0.2850 | 0.3990 | 0.1069 |

Table A. 27. Conditional probability tables -TPM model


Table A. 27 (Continued)

|  |  | 0 O | N |  |  | 50 | $B E A$ | S |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0.0000 \\ & 0 \cdot 1463 \\ & 0.21005 \\ & 0.5000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 0000 \\ & 0.2847 \\ & 0.0000 \\ & 0.5000 \\ & 0.6000 \end{aligned}$ | $\begin{gathered} \text { MAY } \\ 0-5000 \\ 0-2032 \\ 0.0000 \\ 0.0000 \\ 0-2000 \end{gathered}$ | $\begin{aligned} & 0.5000 \\ & 0 \cdot 2439 \\ & 0 \cdot 0000 \\ & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.1219 \\ & 0.7894 \\ & 0.0000 \\ & 0.20000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 0000 \\ & 0 \cdot 0000 \\ & 0 \cdot 3333 \\ & 0 \cdot 2000 \\ & 0 \cdot 3333 \end{aligned}$ | $\begin{aligned} & 0 \cdot 3334 \\ & 0 \cdot 0000 \\ & 0 \cdot 0000 \\ & 0-0000 \\ & 0.3333 \end{aligned}$ | $\begin{aligned} & \text { APRIL } \\ & 0=O C C O \\ & 0=50 \subset 0 \\ & 0.54: 7 \\ & 0=4 C O 0 \\ & 0.00 C C \end{aligned}$ | $\begin{aligned} & 0 \cdot 5000 \\ & 0 \cdot 5000 \\ & 0 \cdot 1250 \\ & 0.0000 \\ & 0.3334 \end{aligned}$ | $\begin{aligned} & 0 \cdot 1666 \\ & 0-0000 \\ & 0 \cdot 0 c 00 \\ & 0.4000 \\ & c \cdot C 000 \end{aligned}$ |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | $\begin{gathered} \text { MAY } \\ 0.2000 \end{gathered}$ |  |  |
| $\begin{aligned} & 0.00 C 0 \\ & 0.1470 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & C \cdot 1840 \end{aligned}$ | $\begin{aligned} & 0.0909 \\ & 0.3750 \end{aligned}$ | 0.9071 0.1470 | $\begin{aligned} & 0 \cdot 0000 \\ & 0 \cdot 1470 \end{aligned}$ | 0.0000 0.0000 | 0.3000 0.0000 | $\begin{aligned} & 0 \cdot 2000 \\ & 0 \cdot 0000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 33 \geq 4 \\ & 0 \cdot 0000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 1666 \\ & 0 \cdot 0000 \end{aligned}$ |
| 0.2400 | 0.0000 | 0.3201 | 0.1099 | 0.2400 | 0.2500 | $0 . C C O C$ | $0 \cdot 3750$ | C. 1250 | C. 2500 |
| 0.0000 | $0 \cdot 2608$ | $0 \cdot 2608$ | $0 \cdot 2176$ | 0.2608 | 0.3125 | 0.0625 | 0.3125 | 0.5000 | c. 3125 |
| $0 \cdot 2500$ | 0.5000 | $0 \cdot 2500$ | 0.0000 | 0.0000 | $0 \cdot 2631$ | C.2105 | 0.2631 | C.2633 | C.0000 |
|  |  | $\begin{gathered} \text { JULY Y } \\ 0.2500 \end{gathered}$ |  |  |  |  | JUNE $0.500 ?$ |  |  |
| $\begin{aligned} & 0 \cdot 2500 \\ & 0.0000 \end{aligned}$ | 0.0000 0.0000 | $\begin{aligned} & 0 \cdot 2500 \\ & 0.500 \mathrm{C} \end{aligned}$ | 0.2500 0.0000 | $0 \cdot 2500$ 0.5000 | 0.0000 0.0000 | 0.0000 0.0000 | $\begin{aligned} & 0 \cdot 5007 \\ & 0 \cdot 6 \in 67 \end{aligned}$ | $\begin{aligned} & 0 \cdot 2500 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 2500 \\ & 0 \cdot 3333 \end{aligned}$ |
| 0.0000 | 0.005 c | $0 \cdot 1843$ | 0.0579 | 0.7578 | 0.0386 | $0 \cdot 1923$ | $0 \cdot 2404$ | 0.3365 | 0.1922 |
| 0.3750 | 0.0000 | 0.3125 | 0.3125 | 0.0000 | $0 \cdot 4827$ | C.0000 | $0.1724$ | $0 \cdot 1725$ | $0.1724$ |
| $0 \cdot 3333$ | $0 \cdot 3333$ | 0.0000 | 0.1666 | $0 \cdot 1657$ | $0 \cdot 3333$ | 0.0000 | $0.3333$ |  |  |
|  |  | AUGUST <br> 0.0000 |  |  |  |  | $\begin{gathered} \text { jury } \\ 0.3334 \end{gathered}$ |  |  |
| 0.2500 0.0000 | 0.0000 0.0000 | $\begin{aligned} & 0.0000 \\ & 0.0000 \end{aligned}$ | 0.2500 0.5000 | $\begin{aligned} & 0 \cdot 5000 \\ & 0.5000 \end{aligned}$ | 0.0000 0.0900 | $\begin{aligned} & 0.0000 \\ & 0.4000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 3334 \\ & 0 \cdot 2000 \end{aligned}$ | $\begin{aligned} & 0.3333 \\ & 0.2000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 3333 \\ & 0.2000 \end{aligned}$ |
| 0.0000 | 0.0000 | $0 \cdot 3077$ | $0 \cdot 6154$ | 0.0769 | 0.2500 | 0.0000 | 0.0000 | 0.5000 | $0 \cdot 25 C 0$ |
| 0.0000 | $0 \cdot 2105$ | 0.2105 | 200000 | 0.5790 | 0.4000 | $0.0000$ | $0.0000$ | $0.2000$ | $0.4000$ |
| 0.4285 | 0.1428 | 0.1428 | $0 \cdot 0: 78$ | 0.2681 | 0.0000 | $0 \cdot 0000$ |  |  |  |
|  |  | SEPTEMBER |  |  |  |  |  |  |  |
| 0.2500 0.0000 | $\begin{aligned} & 0 \cdot 0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.5000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 2500 \\ & 1 \cdot 0000 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.3334 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.1111 \end{aligned}$ | $\begin{aligned} & 0.3334 \\ & 0.5555 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & c \cdot C 000 \end{aligned}$ | $\begin{aligned} & 0.6666 \\ & 0.0060 \end{aligned}$ |
| $0.0000$ | $0.0000$ | $0.0000$ | $0.2580$ | $\begin{aligned} & 1 \cdot 0.00 \\ & 0 \bullet 7420 \end{aligned}$ | 0.0000 | - 0.2500 | 0.0000 | 0.2500 | 0.5000 |
| 0.1750 | 0.0000 | 0.0000 | 0.4250 | 0.4000 | 0.1566 | $0 \cdot 0000$ | $0 \cdot 1667$ | 0.5000 | C. 1667 |
| $0 \cdot 3035$ | $0 \cdot 2857$ | 0.1428 | 0.1250 | 0.1430 | 0.4000 | 0.0000 | 0.0000 | 0.2000 | 0.4000 |

Table A.28. Conditional probability tables - MAPM model


Table A. 28 (Continued)


Table A.29. Conditional probability tables - TVLM model


Table A. 29 (Continued)


Table A. 30. Conditional probability tables - SEM model


| $\begin{aligned} & 0 \bullet 2857 \\ & 0 \bullet 5000 \\ & 0 \bullet 4000 \\ & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 \bullet 2857 \\ & 0 \bullet 0000 \\ & 0 \bullet 0000 \\ & 0.0000 \\ & 0.00000 \end{aligned}$ | $\begin{gathered} \text { DECEMBER } \\ 0.1429 \\ 0.0000 \\ 0.2000 \\ 0.0000 \\ 0.3333 \end{gathered}$ | $\begin{aligned} & 0 \cdot 2857 \\ & 0 \cdot 5000 \\ & 0 \cdot 2000 \\ & 0.6667 \\ & 0.3333 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0: 0000 \\ & 0 \cdot 2000 \\ & 0.3333 \\ & 0.3334 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  | JANUARY |  |  |
| 0.0000 | 0.0000 | O. 5000 | 0.0000 | 0.5000 |
| 0.3333 | $0 \cdot 0000$ | 0.3333 | 0.1667 | 0. 1667 |
| 0. 3750 | $0 \cdot 1250$ | 0.0000 | 0.1250 | 0. 3750 |
| 0.0000 | 0. 2000 | 0.0000 | 0.2000 | 0.6000 |
| 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  |  | FEGRUARY |  |  |
| C. 0000 | 0.000 | 0.0000 | 0.0000 | 1.000 |
| 0.2000 | 0.0000 | 0.4000 | 0.2000 | 0.2000 |
| 0.5714 | 0.1429 | 0.0000 | $0 \cdot 1429$ | $0 \cdot 1428$ |
| 0.0000 | 0.0000 | 0.0000 | 0.4000 | 0.6000 |
| 0.2500 | 0.2500 | 0.0000 | 0.0000 | 0.5000 |
|  |  | MARCH |  |  |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |
| 0.0000 | 0.0000 | 0.5000 | 0.5000 | 0.0000 |
| $0 \cdot 3333$ | 0.1111 | 0.1111 | 0.2222 | 0.2223 |
| $0 \cdot 3333$ | 0.0000 | 0.3333 | 0.0000 | 0.3334 |
| 0.0000 | 0.1429 | 0.0000 | 0.1429 | 0.7142 |
|  |  | APRIL |  |  |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.000 |
| 0.2000 | 0.0000 | $0 \cdot 4000$ | 0.0000 | 0.4000 |
| 0.3750 | 0.125 C | 0.0000 | 0.3750 | $0 \cdot 1250$ |
| 0. 2000 | 0.0000 | 0.2000 | 0.2000 | 0.4000 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |

Table A. 30 (Continued)


C $O R N$


Table A.31. Conditional probability tables, CBT-F model

|  |  | 0 R |  |  |  | 50 | $B E A N$ | S |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0 \bullet 8571 \\ & 0 \bullet 0000 \\ & 6 \bullet 2068 \\ & 0.0000 \\ & 0.0000 \end{aligned}$ |  | $\begin{gathered} \text { DECEMBER } \\ 0.0000 \\ 0.9231 \\ 0.4139 \\ 0.0000 \\ 0.0000 \end{gathered}$ | $\begin{aligned} & 0 \bullet 0000 \\ & 0 \cdot 0000 \\ & 0 \cdot 2069 \\ & 0.3334 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 \bullet 0000 \\ & 0 \bullet 0769 \\ & 0 \cdot 1724 \\ & 0 \bullet 6666 \\ & 1-0000 \end{aligned}$ | $\begin{aligned} & 0.8572 \\ & 0.0 c 00 \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.1428 \\ & 0.50 C 0 \\ & C \cdot C O C O \\ & 0.0000 \\ & 0.0000 \end{aligned}$ | NOVEMBER $0.0 C 00$ 0.2500 0.5000 0.0000 0.0000 | $\begin{aligned} & 0 \cdot 0000 \\ & 0 \cdot 2500 \\ & 0 \cdot 5000 \\ & 0 \cdot 6667 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 0000 \\ & 0.0000 \\ & 0.0000 \\ & 0.333 .3 \\ & 1-0000 \end{aligned}$ |
|  | $0 \cdot 1429$ |  |  |  |  |  |  |  |  |
|  | $0 \cdot 0000$ |  |  |  |  |  |  |  |  |
|  | 0.0000 |  |  |  |  |  |  |  |  |
|  | 0.0000 |  |  |  |  |  |  |  |  |
|  | 0.0000 |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 1.0000 \\ & 0.0000 \\ & 0.0000 \\ & 0.00000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 \bullet 0000 \\ & 0.0770 \\ & 0.0920 \\ & 0.0000 \\ & 0.00000 \end{aligned}$ | $\begin{aligned} & \text { MARCH } \\ & 0.0000 \\ & 0.4615 \\ & 0.4630 \\ & 0.2777 \\ & 0.2926 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.0000 \\ & 0.3333 \\ & 0.7223 \\ & 0.4146 \end{aligned}$ | $\begin{aligned} & 0.000 C \\ & 0.4615 \\ & 0 \bullet 1111 \\ & C .0000 \\ & 0.2928 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0 . \\ & 0 \\ & 0 . \\ & 0 \end{aligned} 1250$ | $\begin{aligned} & 0 \bullet 00 C O \\ & 0 \bullet 000 C \\ & 0 \bullet 1250 \\ & 0 \bullet 0000 \\ & 0.0000 \end{aligned}$ | $\begin{array}{r} \text { JANUARY } \\ 0.3333 \\ 0.7500 \\ 0.1250 \\ 0.6000 \\ 0.0000 \end{array}$ | $\begin{aligned} & 0 \bullet 6667 \\ & 0 \bullet 2500 \\ & 0 \bullet 5000 \\ & 0 \bullet 4000 \\ & 1 \cdot 0000 \end{aligned}$ | $\begin{aligned} & 0 \bullet 0000 \\ & 0-0000 \\ & 0-1250 \\ & 0.0000 \\ & C-0000 \end{aligned}$ |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 0 \bullet 0000 \\ & 0 \bullet 0204 \\ & 0 \cdot 2174 \\ & 0.0000 \\ & 0.2000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 0000 \\ & 0 \cdot 1224 \\ & 0 \cdot 0000 \\ & 0 \cdot 0000 \\ & 0 \cdot 2000 \end{aligned}$ | $\begin{gathered} \text { MAY } \\ 0.5000 \\ 0.6123 \\ 0.7826 \\ 0.5000 \\ 0.2000 \end{gathered}$ | $\begin{aligned} & 0.5000 \\ & 0.0000 \\ & 0 \bullet 0000 \\ & 0.5000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 \bullet 0000 \\ & 0 \bullet 2449 \\ & 0 \because 0000 \\ & 0 \bullet 0000 \\ & 0.4000 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.2500 \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0 \cdot 0000 \\ & 0 \bullet 0000 \\ & 0 \cdot 0000 \\ & 0 \cdot 2000 \\ & 0 \cdot C 0 C 0 \end{aligned}$ | $\begin{aligned} & \text { MARCH } \\ & 0 \cdot 3333 \\ & 0.7500 \\ & 0.7143 \\ & 0.6000 \\ & 1.0 C C C \end{aligned}$ | $\begin{aligned} & C \bullet 6667 \\ & 0 \bullet 0000 \\ & 0 \bullet 2857 \\ & 0 \cdot 2000 \\ & 0 \bullet 0000 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \end{aligned}$ |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| $\begin{aligned} & 0 \bullet 0000 \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.2500 \\ & 0.5000 \\ & 0.0000 \\ & 0.0000 \\ & 0.1666 \end{aligned}$ | $\begin{gathered} \text { JULY } \\ 0.2500 \\ 0.0000 \\ 0.6667 \\ 0.0000 \\ 0.3333 \end{gathered}$ | $\begin{aligned} & 0.2500 \\ & 0 \bullet 0000 \\ & 0 \cdot 1667 \\ & 0.6667 \\ & c \cdot 33 \equiv 3 \end{aligned}$ | $\begin{aligned} & 0 \bullet 2500 \\ & 0 \bullet 5000 \\ & 0 \bullet 1606 \\ & 0 \bullet 3333 \\ & 0 \bullet 1663 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0.000 C \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & 0.0000 \\ & 0 \cdot 0000 \\ & 0 \cdot 0000 \\ & 0 \cdot 0000 \\ & 0.0000 \end{aligned}$ | MAY |  | $\begin{aligned} & C \cdot 3333 \\ & 0 \cdot 0000 \\ & 0.1250 \\ & C \cdot 3333 \\ & 0.2500 \end{aligned}$ |
|  |  |  |  |  |  |  | $0 \cdot 1.667$ | $0 \cdot 5000$ |  |
|  |  |  |  |  |  |  | 0.0000 | $0 \cdot 0000$ |  |
|  |  |  |  |  |  |  | $0 \cdot 2500$ | 0-6250 |  |
|  |  |  |  |  |  |  | $0 \cdot 3333$ | $0 \cdot 3334$ |  |
|  |  |  |  |  |  |  | $0 \cdot \mathrm{COOO}$ | $0 \cdot 7500$ |  |
|  |  |  |  |  | $\begin{aligned} & 0.0000 \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \end{aligned}$ | $\begin{aligned} & C .0000 \\ & 0 \because 0000 \\ & 0.0000 \\ & 0.0000 \\ & C .0 C 00 \end{aligned}$ | $\begin{aligned} & \text { JULY } \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \\ & 0.0000 \\ & C .2500 \end{aligned}$ | $\begin{aligned} & 0.6667 \\ & 0.8000 \\ & 0.7500 \\ & 0.8000 \\ & 0.6875 \end{aligned}$ | $\begin{aligned} & 0 \bullet 3333 \\ & 0 \bullet 2000 \\ & 0 \bullet 2500 \\ & 0 \bullet 2000 \\ & c \cdot 0 \in 25 \end{aligned}$ |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

Table A.32. Data-prior probability matrices

| TPM-MODEL |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0 \cdot 3333$ | 0.1429 | 0.1429 | $0 \cdot 0476$ | 0.3333 | $0 \cdot 3333$ | $0 \cdot 0476$ | $0 \cdot 2857$ | 0.1905 | 0-1429 |
| $0 \cdot 0455$ | 0.2727 | 0.3182 | 0.2727 | 0.0909 | $0 \cdot 2857$ | $0 \cdot 0952$ | $0 \cdot 2857$ | 0.1905 | $0 \cdot 1429$ |
| 0.0455 | $0 \cdot 2273$ | 0.2726 | $0 \cdot 2273$ | 0.2273 | $0 \cdot 1364$ | 0.1818 | $0 \cdot 3636$ | 0.2727 | C.0455 |
| 0.0000 | $0.1364$ | 0.4091 | 0.1364 | 0.3181 | 0.1818 | 0.0455 | 0.4091 | 0.2727 | $0 \cdot 0909$ |
| 0.0000 | $0 \cdot 2727$ | $0 \cdot 3181$ | 0.1819 | $0 \cdot 2273$ | 0. 1818 | 0.0909 | 0.4091 | 0.2273 | 0.0909 |
| 0.1364 | 0.2727 | $0 \cdot 2273$ | $0 \cdot 1364$ | $0 \cdot 2272$ | 0.1818 | $0 \cdot 1364$ | $0 \cdot 2727$ | 0.2727 | $0 \cdot 1364$ |
| $0 \cdot 1364$ | $0 \cdot 1818$ | $0 \cdot 3182$ | 0.2272 | $0 \cdot 1364$ | 0.1818 | 0-1136 | $0 \cdot 2955$ | $0 \cdot 2273$ | $0 \cdot 1818$ |
| 0.1818 | 0.0909 | 0.2272 | 0.1364 | $0 \cdot 3637$ | $0 \cdot 1818$ | 0.0909 | $0 \cdot 3182$ | 0.2273 | 0.1815 |
| 0.1818 | 0.0909 | 0.1818 | 0.1818 | $0 \cdot 3637$ | 0.1364 | $0 \cdot 1364$ | $0 \cdot 2273$ | 0.2273 | 0.2726 |
| $0 \cdot 1818$ | 0.0909 | 0.0909 | $0 \cdot 2727$ | $0 \cdot 3637$ | $0 \cdot 1818$ | $0 \cdot 0455$ | $0 \cdot 2273$ | 0.2273 | C. 3181 |
| MAPM-MODEL |  |  |  |  |  |  |  |  |  |
| $0 \cdot 1364$ | $0 \cdot 3182$ | 0.0909 | $0 \cdot 1818$ | O-2727 | O. 1364 | $0 \cdot 3182$ | $0 \cdot 0455$ | 0.0909 | 0-4090 |
| $0 \cdot 2273$ | 0.1818 | 0.1364 | $0 \cdot 1364$ | 0.3181 | $0 \cdot 1818$ | 0.2727 | 0.0455 | $0 \cdot 1818$ | $0 \cdot 3182$ |
| $0 \cdot 1364$ | 0.2273 | $0 \cdot 1364$ | O-1818 | $0 \cdot 3181$ | $0 \cdot 3182$ | $0 \cdot 1364$ | $0 \cdot 0000$ | 0.0455 | $0 \cdot 4999$ |
| $0 \cdot 1364$ | 0.1818 | $0 \cdot 1364$ | $0 \cdot 1818$ | $0 \cdot 3636$ | 0.2727 | $0 \cdot 1818$ | 0.0000 | 0-1364 | $0 \cdot 4091$ |
| $0 \cdot 2273$ | 0.1364 | $0 \cdot 1364$ | $0 \cdot 1364$ | 0.3635 | $0 \cdot 3182$ | 0.1364 | 0.0000 | $0 \cdot 1364$ | $0 \cdot 4090$ |
| $0 \cdot 3182$ | 0.1818 | $0 \cdot 0455$ | 0.0909 | $0 \cdot 3636$ | $0 \cdot 2727$ | $0 \cdot 1364$ | $0 \cdot 1364$ | $0 \cdot 1364$ | $0 \cdot 3181$ |
| $0 \cdot 3182$ | $0 \cdot 1364$ | 0.1364 | $0 \cdot 0455$ | $0 \cdot 3635$ | $0 \cdot 1818$ | $0 \cdot 2273$ | $0 \cdot 0909$ | $0 \cdot 2273$ | 0.2727 |
| 0.2273 | 0.2273 | 0.0455 | 0.1364 | $0 \cdot 3635$ | $0 \cdot 1818$ | $0 \cdot 1818$ | $0 \cdot 1818$ | $0 \cdot 1364$ | $0 \cdot 3182$ |
| 2. 2364 | 0.1364 | 0.2273 | 0.0909 | $0 \cdot 4090$ | 0.0909 | $0 \cdot 1818$ | $0 \cdot 1364$ | $0 \cdot 1818$ | $0 \cdot 4091$ |
| 0.0909 | $0 \cdot 1364$ | $0 \cdot 1364$ | 0.1364 | 0.4999 | 0.0455 | $0 \cdot 1818$ | 0.2727 | $0 \cdot 1364$ | $0 \cdot 3636$ |
| TVLM-MODEL 0 - 0 , 063 |  |  |  |  |  |  |  |  |  |
| $0 \cdot 3182$ |  | $0.2727$ | $0 \cdot 2727$ | $0 \cdot 1364$ | $\begin{aligned} & 0 \cdot 1364 \\ & 0 \cdot 1812 \end{aligned}$ | $0.2273$ | $0.1818$ | $0 \cdot 3182$ | $0 \cdot 1363$ |
| $0.2727$ | $0.0455$ | $0.1818$ | $0.1364$ | $0 \cdot 3636$ | $0.1818$ | $0 \cdot 1364$ | $0 \cdot 3182$ | $0 \cdot 1818$ | $0.1818$ |
| $0.2727$ | $0.0455$ | $0 \cdot 2273$ | $0 \cdot 0455$ | $0 \cdot 4090$ | $0.2727$ | 0.0909 | $0 \cdot 1364$ | 0.1818 | $0 \cdot 3182$ |
| 0.2273 | 0.0909 | 0.0909 | $0 \cdot 1818$ | 0.4092 | $0 \cdot 2273$ | $0 \cdot 1818$ | $0 \cdot 1818$ | 0.0909 | $0 \cdot 3182$ |
| $0 \cdot 2273$ | 0.0455 | $0 \cdot 1364$ | $0 \cdot 1818$ | 0.4090 | 0.2273 | $0 \cdot 1364$ | $0 \cdot 1818$ | 0.2273 | $0 \cdot 2272$ |
| 0.2273 | 0.0455 | 0.1364 | 0.1818 | $0 \cdot 4090$ | 0.3182 | $0 \cdot 0455$ | $0 \cdot 2273$ | $0 \cdot 2273$ | $0 \cdot 1817$ |
| $0 \cdot 2273$ | 0.0455 | 0.0909 | $0 \cdot 2727$ | $0 \cdot 3636$ | $0 \cdot 1818$ | 0.2273 | 0.1364 | $0 \cdot 3182$ | 0.1363 |
| 0.2727 | $0 \cdot 0000$ | $0 \cdot 1364$ | $0 \cdot 1364$ | $0 \cdot 4545$ | $0 \cdot 2727$ | $0 \cdot 1818$ | $0 \cdot 3182$ | $0.1818$ | $0.0455$ |
| $0 \cdot 1818$ | 0.0455 | 0.1818 | $0 \cdot 1364$ | 0.4545 | $0 \cdot 0908$ | $0 \cdot 2273$ | $0 \cdot 2273$ | $0 \cdot 2273$ | $0.2273$ |

Table A. 32. (Continued)


C $C \quad R \quad N$

SEM- MODEL



[^0]:    ${ }^{1}$ CC is the prevailing cost of capital in the sector and region under consideration.

[^1]:    ${ }^{1}$ Any persistent discrepancy arising between the market prices and those prices received by the $f^{\text {th }}$ farmer must also be included here.

[^2]:    ${ }^{1}$ We follow closely P. E. Green and D. S. Tull (14, Chapter 3).

[^3]:    $a_{\text {The predictions are based on the prices received by farmers in }}$ Iowa as reported by the USDA, ERS.

[^4]:    ${ }^{l_{C}}$ and S.B. are used to abbreviate corn and soybeans, respectively.

[^5]:    ${ }^{l_{T h e ~}}$ SEM model fails to pass the tests of equal variance in the month of August at $95 \%$ level of significance, however, the margin is very small; calculated $F=2.0441$, tabulated $\mathrm{F}=2.0400$.

[^6]:    ${ }^{1}$ M.S. is used to abbreviate Marketing Season.

